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A Tractable Truthful Profit Maximization Mechanism Design with Autonomous Agents

Mina Montazeri, Hamed Kebriaei*, *Senior Member, IEEE,* and Babak N. Araabi

***Abstract*— Task allocation is a crucial process in modern systems, but it is often challenged by incomplete information about the utilities of participating agents. In this paper, we propose a new profit maximization mechanism for the task allocation problem, where the task publisher seeks an optimal incentive function to maximize its own profit and simultaneously ensure the truthful announcing of the agent's private information (type) and its participation in the task, while an autonomous agent aims at maximizing its own utility function by deciding on its participation level and announced type. Our mechanism stands out from the classical contract theory-based truthful mechanisms as it empowers agents to make their own decisions about their level of involvement, making it more practical for many real-world task allocation scenarios. It has been proven that by considering a linear form of incentive function consisting of two decision functions for the task publisher the mechanism's goals are met. The proposed mechanism is initially modeled as a non-convex functional optimization with the double continuum of constraints, nevertheless, we demonstrate that by deriving an equivalent form of the incentive constraints, it can be reformulated as a tractable convex optimal control problem. Further, we propose a numerical algorithm to obtain the solution.**

***Index Terms*— Task allocation, profit maximization mechanism, functional optimization, incomplete information.**

I. Introduction

Task allocation is an essential aspect of many systems, including supply chain management [1], transportation [2], and distributed computing [3]. It involves the assignment of tasks to the agents based on their capabilities, with the goal of maximizing efficiency and achieving the desired outcome. However, in most task allocation applications, the task publisher faces several challenges like incomplete information about the agents' utilities, and the autonomy of the agents in deciding on their own participation levels. These challenges hinder optimal decision making in the task allocation problem.

One approach that has been proposed in the literature for addressing this problem is the Bayesian Stackelberg game (BSG) e.g. in power allocation problems [4], demand response [5], and crowdsensing [6]. In a BSG, the task publisher (leader) is uncertain about the agent's type which is a parameter (or some parameters) in the agent's (follower) objective function [7]. Thus, the leader maximizes its own expected payoff with respect to the distribution of the follower agent's type, subject to the best response of the follower. However, in BSG, it is assumed that an agent voluntarily participates in the game. This is not practical in applications like task allocation, where the cost incurred by doing the task may result in a negative payoff for the agent. Further, the optimal strategy of the task publisher in BSG is obtained in the average sense with respect to the distribution of the agent's type, and hence, the agent receives an incentive function from the task publisher which is not necessarily designed in accordance with its actual type. Economic theory provides an elegant tool called "mechanism design" to address such challenges.

Mechanism design offers a framework, especially for task allocation problems under asymmetric information in which, the task publisher is not aware of the agents' private information [8]. There are two kinds of mechanisms in the literature: direct mechanisms and indirect mechanisms. In the direct revelation mechanism, each agent is asked to announce its private information. While, in the latter, agents don't announce their private information directly and agents' preferences can be observed only indirectly through their decisions.

In a direct revelation mechanism for the task/resource allocation problem, the only action available to the agents is to announce their types. In this case, the task publisher allocates a participation level and the corresponding incentive reward to each agent as the functions of its announced type, in order to achieve three objectives, simultaneously: motivate agents to participate in the task, ensure truthful announcing of the private information of agents, and maximize the task publisher's utility (or maximize the social welfare) [9]. The direct mechanism may or may not induce a game among the agents. The latter is called also contract theory [10], while VCG is a well-known example of the former. Some direct mechanisms also consider further properties like (weak) budget balance [11] or multi-dimensional private information [12]. However, in numerous scenarios of resource/task allocation, such as federated learning and crowdsensing, there exists a distinct inclination among agents to independently manage their participation levels. The utilization of direct mechanisms, particularly within the context of contract theory, can inadvertently introduce certain unrealistic assumptions in these domains [9]. In such contexts, the implementation of indirect mechanisms presents a more pragmatic and suitable alternative. There are several pieces of research in the literature that design indirect mechanisms for resource/task allocation problems. Most of the proposed mechanisms satisfy criteria, such as Nash optimality, budget balance, or individual rationality [13], [14] while some of them provide an algorithm for their mechanism to reach the equilibrium point [15], [16]. While the agents in these indirect mechanisms are not asked to announce their private information, this kind of mechanism is applicable for social welfare maximization, assuming the designer is not a profit maximizer.

In this paper, we propose a new profit maximization mechanism for the task allocation problem to achieve the following goals (i) Maximize the task publisher's utility function (as in direct mechanisms, contract theory, and Stackelberg game and unlike indirect mechanisms) (ii) Let autonomous agents decide on their own participation level (as in Stackelberg game, and indirect mechanisms and unlike direct mechanisms) (iii) Guarantee the participation of agents and also truthful announcing of agent's type in the mechanism (as in direct mechanisms and unlike indirect mechanisms, and Stackelberg games). Achieving these goals is essential for many task allocation problems such as federated learning [9], crowdsensing, and

*Corresponding author: Hamed Kebriaei.
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Mina Montazeri, Hamed Kebriaei and Babak N. Araabi are with the School of ECE, College of Engineering, University of Tehran, Tehran, Iran. Emails: (mina.montazeri@ut.ac.ir, kebriaei@ut.ac.ir, araabi@ut.ac.ir).
Mina Montazeri is also with the Urban Energy Systems Laboratory, Swiss Federal Laboratories for Materials Science and Technology, Dübendorf, Switzerland. Email: (mina.montazeri@empa.ch).
Hamed Kebriaei is also with the School of Computer Science, Institute for Research in Fundamental Sciences (IPM) P.O. Box 19395-5746, Tehran, Iran.

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crowdsourcing [6], [17]. To the best of the authors' knowledge, this paper introduces the first truthful profit maximization mechanism that achieves these three goals, simultaneously.

In the proposed mechanism, the task publisher seeks an optimal reward as a function of the agent's announced type and participation level, in order to maximize its own profit. After the reward function is imposed on the agents, each autonomous agent determines its optimal announced type and participation level. Therefore, the optimization problem of the task publisher is subject to some constraints which are: the agents' best response to the reward function, non-negative profit making of the agents out of participation in the task, and making the truthful announcing of the type as the best strategy of the agents to the reward function. We show that, by using a linear form of the incentive reward function, including two decision functions, the task publisher achieves these goals, simultaneously. The main contributions of the paper are as follows:

- We propose a new truthful mechanism for the task allocation problem that allows autonomous agents to selfishly decide on their own participation levels while ensuring the truthful announcing of the agent's private information (type), its participation in the task, and maximizing the task publisher's profit.
- We prove that by considering a linear form of the incentive reward function, including two decision functions for the task publisher, all the mentioned properties can be satisfied.
- We prove that by introducing a relation between the decision functions of the task publisher, the main non-convex functional optimization problem can be reformulated to an equivalent tractable convex optimal control problem.

Notation: The symbols $\mathbb{R}$, $\mathbb{R}^+$, and $\mathbb{R}^n$ denote as real numbers, positive real numbers, and the set of n-dimensional real column vectors, respectively; $\mathbf{1}$ denotes as the all-ones vector. For a given vector or matrix X, X^T denotes as its transpose. Given a set $\mathbb{U}$ and a point y, the projection of y onto $\mathbb{U}$, denoted by $P_U(y) \in \mathbb{U}$ satisfies $\|y - P_U(y)\| \leq \|y - v\| \quad \forall v \in \mathbb{U}$. $\left(\frac{\partial f}{\partial x}\right)_{x^*} = \frac{\partial f(x)}{\partial x}\Big|_{x=x^*}$ denotes as the partial derivative of $f(x)$ with respect to x at point $x = x^*$.

II. PROFIT MAXIMIZATION MECHANISM

We consider a task allocation problem comprising two main parties: the task publisher and the autonomous agents. Inspired by [18], [19], the utility function of the agent is formulated as follows

$$U(\theta, x, R(x, \hat{\theta})) = S(x, \theta) + R(x, \hat{\theta}) \tag{1}$$

where $x \in \mathbb{R}^+$ is the level of participation of the agent in the task, $\theta \in \Theta$ with $\Theta = [\underline{\theta}, \bar{\theta}]$ and $\underline{\theta}, \bar{\theta} > 0$ represents the level of the agent's willingness to participate in the task which is the private information of the agent and is treated as its type and $\hat{\theta} \in \Theta$ is the announced type which is not necessarily equal to the actual type (i.e. θ) as agents may have the incentive to announce their type incorrectly if it makes more profit for them. Although neither the task publisher nor other agents know the agent's type, its cumulative distribution $F(\theta)$ is common knowledge. Function $R(x, \hat{\theta}) : \mathbb{R}^+ \times \Theta \to \mathbb{R}^+$ is the incentive reward function that each agent receives from the task publisher. $S(x, \theta) = \theta\pi(x) - px$ is the satisfaction function of the agent from participating in the task and it includes two parts: the first term, i.e., $\theta\pi(x) : \mathbb{R}^+ \times \Theta \to \mathbb{R}$, represents the revenue of the agent obtained from participation in the task and the second term, i.e., px, represents the cost incurred by doing tasks with $p > 0$ as the linear cost coefficient or marginal cost.

The task publisher's utility function is defined as follows

$$V(x, R(x, \hat{\theta})) = g(x) - R(x, \hat{\theta}). \tag{2}$$

where $g : \mathbb{R}^+ \to \mathbb{R}$ is the task publisher's revenue function from agents' participation in the task. The information flow between the task publisher and autonomous agents is depicted in Figure 1. In this

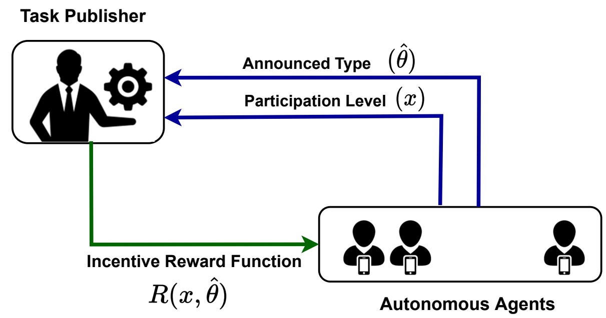

Fig. 1. The information flow between task publisher and agents.

study, we consider settings without externalities, where the payoffs received by each agent only depend on their level of participation and there is no game among the agents.

Remark 1: The proposed model can also be extended to handle n independent tasks with $U(\theta, x, R(x, \hat{\theta})) = \theta^T\pi(x) - p^Tx + \mathbf{1}^TR(x, \hat{\theta})$ and $V(x, R(x, \hat{\theta})) = \mathbf{1}^Tg(x) - \mathbf{1}^TR(x, \hat{\theta})$, where $x = (x_1, \ldots, x_n)^\top \in \mathbb{R}^n$, $\theta = (\theta_1, \ldots, \theta_n)^\top \in \mathbb{R}^n$ and similarly $\hat{\theta}, \pi(x), g(x), p$ and $R(x, \hat{\theta})$ as vectors of $\hat{\theta}_i, \pi_i(x_i), g_i(x_i), p_i, R_i(x_i, \theta_i)$, respectively.
As is customary in the literature [19], we assert the following assumption on functions g and π.

Assumption 1: Functions $\pi(\cdot)$ and $g(\cdot)$ are non-decreasing and strongly concave.

In the proposed profit maximization mechanism, the task publisher adopts the following form of the reward function.

$$R(x, \hat{\theta}) \equiv \alpha(\hat{\theta})x + \beta(\hat{\theta}) \tag{3}$$

where $\alpha(\hat{\theta})$ and $\beta(\hat{\theta})$ are both decision functions of the task publisher, which respectively represent the reward factor for the participation level paid by the task publisher and the bias reward from the task publisher to the agent. Replacing (3) into (1), the utility of the agent can be redefined as follows:

$$\tilde{U}(\theta, x, \alpha(\hat{\theta}), \beta(\hat{\theta})) = U(\theta, x, \alpha(\hat{\theta})x + \beta(\hat{\theta})) \tag{4}$$

We show that this linear form of incentive reward function is rich enough to enable the task publisher to maximize its profit while satisfying three constraints simultaneously: (i) react optimally to the best response of the agents, (ii) motivate agents to participate in the task, (iii) ensure truthful announcing of the agent's type. The formal definitions of the last two goals are presented in the fallowing.

Definition 1: A mechanism is Individually Rational (IR), if the agent's utility is non-negative by truthful announcing of the type, i.e. $\hat{\theta} = \theta$. Specifically, the mechanism is IR if

$$\tilde{U}(\theta, x, \alpha(\theta), \beta(\theta)) \geq 0. \tag{5}$$

Definition 2: A mechanism is Incentive Compatible (IC) if the agent achieves maximum utility by truthful announcing of the type, i.e. $\hat{\theta} = \theta$. Specifically, the mechanism is IC if

$$\tilde{U}(\theta, x, \alpha(\theta), \beta(\theta)) \geq \tilde{U}(\theta, x, \alpha(\hat{\theta}), \beta(\hat{\theta})) \quad \forall \theta, \hat{\theta} \in \Theta. \tag{6}$$

The task publisher's goal is to maximize its utility subject to the agent's best response, the Individual Rationality (IR), and the Incentive Compatibility (IC) constraints. Therefore, the optimal profit maximization mechanism can be obtained by solving the following maximization problem.

$$\max_{\alpha(\hat{\theta}),\beta(\hat{\theta})} \mathbb{E}_\theta[V(\chi,\alpha(\hat{\theta}),\beta(\hat{\theta}))] \tag{7a}$$

$$s.t.\ \ \chi(\theta,\alpha(\hat{\theta}),\beta(\hat{\theta})) = \underset{x}{\operatorname{argmax}}\, U(\theta,x,\alpha(\hat{\theta}),\beta(\hat{\theta})) \tag{7b}$$

$$\tilde{U}(\theta,\chi,\alpha(\theta),\beta(\theta)) \geq 0, \qquad \forall\theta\in\Theta \tag{7c}$$

$$\tilde{U}(\theta,\chi,\alpha(\theta),\beta(\theta)) \geq \tilde{U}(\theta,\chi,\alpha(\hat{\theta}),\beta(\hat{\theta})),\ \ \forall\theta,\hat{\theta}\in\Theta. \tag{7d}$$

Based on the constraints (7b) and (7d), the task publisher takes into account the best response of the agents on the variables x and $\hat{\theta}$, respectively and further, according to (7d), the reward function is designed such that the optimal announced type by the agents, is equal to their actual type θ.

For ease of notation, we set $\chi = \chi(\theta,\alpha(\hat{\theta}),\beta(\hat{\theta}))$ when we do not want to indicate that χ is a function of θ, $\alpha(\hat{\theta})$, and $\beta(\hat{\theta})$.

III. TRACTABLE REFORMULATION OF THE MECHANISM

Solving (7) is not straightforward due to the nonconvex double continuum constraint imposed by the IC constraint [20]. To tackle this issue, in this section, we follow a multi-step approach to obtain an *"equivalent tractable reformulation"* for optimization (7).

Proposition 1: Constraint (7b) is equivalent to

$$\chi(\theta,\alpha(\hat{\theta}),\beta(\hat{\theta})) = \Gamma(\frac{p-\alpha(\hat{\theta})}{\theta}) \tag{8}$$

where function $\Gamma(.)$ is the inverse function of $\left(\frac{\partial\pi}{\partial x}\right)_\chi$.

Proof: In (7b), χ is a critical point for $\tilde{U}(\theta,x,\alpha(\hat{\theta}),\beta(\hat{\theta}))$ over x and from Assumption 1, $\tilde{U}$ is a strictly concave function in x. Thus, using the first-order optimality condition we have

$$\left(\frac{\partial\pi}{\partial x}\right)_\chi = \frac{p-\alpha(\hat{\theta})}{\theta}. \tag{9}$$

From Assumption 1 $\frac{\partial\pi(x)}{\partial x}$ is a strictly monotone function. Thus, we can define the function $\Gamma(.)$ as the inverse function of $\left(\frac{\partial\pi}{\partial x}\right)_\chi$, which gives $\chi(\theta,\alpha(\hat{\theta}),\beta(\hat{\theta})) = \Gamma(\frac{p-\alpha(\hat{\theta})}{\theta})$. ■

From Proposition 1, it comes out that function χ is not function of β and hence, to simplify the notation we define $\hat{\chi}$ as follows:

$$\hat{\chi}(\theta,\alpha(\hat{\theta})) = \chi(\theta,\alpha(\hat{\theta}),\beta(\hat{\theta})) \tag{10}$$

Remark 2: According to Assumption 1, $\frac{\partial\pi(x)}{\partial x}$ is a decreasing function. Thus, function Γ as an inverse function of $\left(\frac{\partial\pi}{\partial x}\right)_{\hat{\chi}}$, is decreasing with respect to $\frac{p-\alpha(\hat{\theta})}{\theta}$. Hence 1) $\frac{\partial\hat{\chi}}{\partial\theta} > 0$, and 2) $\frac{\partial\hat{\chi}}{\partial\alpha} > 0$.

Proposition 2 presents an alternative formulation for the IR constraint, i.e. constraint (7c), within the setting of the optimization problem (7).

Proposition 2: Constraint (7c) in the context of the optimization problem (7) is equivalent to the following constraint:

$$\tilde{U}(\underline{\theta},\hat{\chi},\alpha(\underline{\theta}),\beta(\underline{\theta})) = 0, \tag{11}$$

which means that satisfying the IR constraint for each $\theta\in\Theta$ is equivalent to satisfying IR constraint in the equality mode for $\underline{\theta}$.

Proof: By substituting $\hat{\chi}$ from (8) and (10) in U, and differentiating U with respect to agent's actual type, i.e. θ, we have

$$\frac{d\tilde{U}(\theta,\hat{\chi},\alpha(\hat{\theta}),\beta(\hat{\theta}))}{d\theta} = \left(\frac{\partial\tilde{U}}{\partial x}\right)_{\hat{\chi}}\frac{\partial\hat{\chi}(\theta,\alpha(\hat{\theta}))}{\partial\theta} + \pi(\hat{\chi}). \tag{12}$$

The first term of (12) equals zero due to the first-order condition, i.e. $\left(\frac{\partial\tilde{U}}{\partial x}\right)_{\hat{\chi}} = 0$, and hence we have

$$\frac{d\tilde{U}(\theta,\hat{\chi},\alpha(\hat{\theta}),\beta(\hat{\theta}))}{d\theta} = \pi(\hat{\chi}) \geq 0. \tag{13}$$

Let us consider the following inequality for the agent with type $\tilde{\theta}\in\Theta$

$$\begin{aligned}&\tilde{U}(\tilde{\theta},\hat{\chi}(\tilde{\theta},\alpha(\tilde{\theta})),\alpha(\tilde{\theta}),\beta(\tilde{\theta})) \geq \tilde{U}(\tilde{\theta},\hat{\chi}(\tilde{\theta},\alpha(\underline{\theta})),\alpha(\underline{\theta}),\beta(\underline{\theta}))\\ &\geq \tilde{U}(\underline{\theta},\hat{\chi}(\underline{\theta},\alpha(\underline{\theta})),\alpha(\underline{\theta}),\beta(\underline{\theta}))\end{aligned} \tag{14}$$

where the first inequality holds from IC constraint (7d) for $\hat{\theta} = \underline{\theta}$, and the second inequality follows from (13). Thus, it follows $\tilde{U}(\tilde{\theta},\hat{\chi}(\tilde{\theta},\alpha(\tilde{\theta})),\alpha(\tilde{\theta}),\beta(\tilde{\theta})) \geq \tilde{U}(\underline{\theta},\hat{\chi}(\underline{\theta},\alpha(\underline{\theta})),\alpha(\underline{\theta}),\beta(\underline{\theta}))$. Hence, if the constraint (7c) is satisfied for $\theta = \underline{\theta}$ and constraint (7d) is satisfied for all $\theta\in\Theta$, then IR constraint (7c) is satisfied for every $\theta > \underline{\theta}$. To complete the proof, it must be shown that IR constraint for $\theta = \underline{\theta}$ is binding. If IR constraint for $\theta = \underline{\theta}$ is not binding, the value of $\beta(\tilde{\theta})$ can be decreased by a sufficiently small $\epsilon > 0$ for all $\tilde{\theta}\in\Theta$ such that, the task publisher's utility increases while both IR constraint for $\theta = \underline{\theta}$ and the IC constraint for all θ are still satisfied. This contradicts with optimality of solution. ■

Next, the optimization (7) is reformulated by introducing a relation between decision functions $\alpha(\hat{\theta})$ and $\beta(\hat{\theta})$. This results in removing the non-convex IC constraints.

Theorem 1: Optimization (7) is equivalent to the following optimization problem.

$$\max_{\alpha(\hat{\theta}),\beta(\hat{\theta})} \mathbb{E}_\theta[V(\hat{\chi},\alpha(\hat{\theta}),\beta(\hat{\theta}))] \tag{15a}$$

$$s.t.\ \ \dot{\alpha}(\hat{\theta}) \geq 0 \tag{15b}$$

$$\beta(\hat{\theta}) = \int_{\underline{\theta}}^{\hat{\theta}} [K_\theta(\hat{\chi}(\theta,\alpha(\hat{\theta})),\theta,\hat{\theta})]\Big|_{\theta=\hat{\theta}} d\hat{\theta} - K(\hat{\chi}(\theta,\alpha(\hat{\theta})),\theta,\hat{\theta})\Big|_{\theta=\hat{\theta}} \tag{15c}$$

$$\hat{\chi}(\theta,\alpha(\hat{\theta})) = \Gamma(\frac{p-\alpha(\hat{\theta})}{\theta}) \tag{15d}$$

where $K(\hat{\chi}(\theta,\alpha(\hat{\theta})),\theta,\alpha(\hat{\theta})) \equiv \alpha(\hat{\theta})\hat{\chi}(\theta,\alpha(\hat{\theta})) + \theta\pi(\hat{\chi}(\theta,\alpha(\hat{\theta}))) - p\hat{\chi}(\theta,\alpha(\hat{\theta}))$ and K_θ defines the derivation of K with respect to the actual type of agent, i.e. θ. Also, $\dot{\alpha}(\hat{\theta})$ signifies the derivative of $\alpha(\hat{\theta})$ with respect to $\hat{\theta}$.

Proof: To show the equivalency of optimization problems (7) and (15), it suffices to prove that for any optimal solution to problem (7), there exists a solution to problem (15) with the same objective value and visa versa. We prove this theorem in two steps. First, we show that given a solution to (15), we can find a corresponding solution to (7) with the same objective value. By considering the definition of agent's utility in (1), K can be rewritten as

$$K(\hat{\chi}(\theta,\alpha(\hat{\theta})),\theta,\alpha(\hat{\theta})) = \tilde{U}(\theta,\hat{\chi}(\theta,\alpha(\hat{\theta})),\alpha(\hat{\theta}),\beta(\hat{\theta})) - \beta(\hat{\theta}). \tag{16}$$

By derivation from (16) with respect to θ, we have

$$\frac{dK}{d\theta} = \left(\frac{\partial\tilde{U}}{\partial x}\right)_{\hat{\chi}}\frac{\partial\hat{\chi}}{\partial\theta} + \frac{\partial\tilde{U}}{\partial\theta} - \frac{\partial\beta(\hat{\theta})}{\partial\theta}. \tag{17}$$

Since β is not a function of θ and $\left(\frac{\partial\tilde{U}}{\partial x}\right)_{\hat{\chi}} = 0$, we obtain $\frac{dK}{d\theta} = \pi(\hat{\chi})$. Using this and replacing β from constraint (15c) into IC constraints (7d) for an agent with type $\tilde{\theta}$, (7d) can be rewritten as

$$\begin{aligned}&\alpha(\tilde{\theta})\hat{\chi}(\tilde{\theta},\alpha(\tilde{\theta})) + \int_{\underline{\theta}}^{\tilde{\theta}}\pi(\hat{\chi}(y,\alpha(y)))dy - \alpha(\tilde{\theta})\hat{\chi}(\tilde{\theta},\alpha(\tilde{\theta})) -\\ &\tilde{\theta}\pi(\hat{\chi}(\tilde{\theta},\alpha(\tilde{\theta}))) + p\hat{\chi}(\tilde{\theta},\alpha(\tilde{\theta})) + \tilde{\theta}\pi(\hat{\chi}(\tilde{\theta},\alpha(\tilde{\theta}))) - p\hat{\chi}(\tilde{\theta},\alpha(\tilde{\theta}))\\ &\geq \alpha(\check{\theta})\hat{\chi}(\tilde{\theta},\alpha(\check{\theta})) + \int_{\underline{\theta}}^{\check{\theta}}\pi(\hat{\chi}(y,\alpha(y)))dy - \alpha(\check{\theta})\hat{\chi}(\check{\theta},\alpha(\check{\theta})) -\\ &\check{\theta}\pi(\hat{\chi}(\check{\theta},\alpha(\check{\theta}))) + p\hat{\chi}(\check{\theta},\alpha(\check{\theta})) + \tilde{\theta}\pi(\hat{\chi}(\tilde{\theta},\alpha(\check{\theta}))) - p\hat{\chi}(\tilde{\theta},\alpha(\check{\theta}))\end{aligned} \tag{18}$$

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where $\check{\theta} \in \Theta$ is the arbitrary announced type. By adding and subtracting $\beta(\check{\theta})$ to the left side of (18) and simplifying it, we have

$$\int_{\check{\theta}}^{\tilde{\theta}} \pi(\hat{\chi}(y,\alpha(y)))dy \geq \tilde{U}(\tilde{\theta},\hat{\chi}(\tilde{\theta},\alpha(\check{\theta})),\alpha(\check{\theta}),\beta(\check{\theta})) - \tilde{U}(\check{\theta},\hat{\chi}(\check{\theta},\alpha(\check{\theta})),\alpha(\check{\theta}),\beta(\check{\theta})). \tag{19}$$

Thus, by considering (13) we have

$$\int_{\check{\theta}}^{\tilde{\theta}} [\frac{d\tilde{U}(\theta,\hat{\chi}(\theta,\alpha(y)),\alpha(y),\beta(y))}{d\theta}\Big|_{\theta=y}]dy \geq \tilde{U}(\tilde{\theta},\hat{\chi}(\tilde{\theta},\alpha(\check{\theta})),\alpha(\check{\theta}),\beta(\check{\theta})) - \tilde{U}(\check{\theta},\hat{\chi}(\check{\theta},\alpha(\check{\theta})),\alpha(\check{\theta}),\beta(\check{\theta})). \tag{20}$$

Next, we show that (20) which is equivalent to IC constraint holds true. By derivation from (13) with respect to $\hat{\theta}$, we have

$$\frac{d^2\tilde{U}(\theta,\hat{\chi}(\theta,\alpha(\hat{\theta})),\alpha(\hat{\theta}),\beta(\hat{\theta}))}{d\hat{\theta}d\theta} = \frac{d\pi}{d\hat{\theta}} = (\frac{\partial\pi}{\partial x})_{\hat{\chi}} \times \frac{\partial\hat{\chi}}{\partial\alpha} \times \frac{\partial\alpha}{\partial\hat{\theta}} \geq 0. \tag{21}$$

Since $\frac{\partial\pi}{\partial x}, \frac{\partial\hat{\chi}}{\partial\alpha}$ and $\frac{\partial\alpha}{\partial\theta}$ are positive as the results of Assumption 1, Remark 2, and constraint (15b), respectively, (18) holds true. Hence, if $\tilde{\theta} > \check{\theta}$, we have

$$\int_{\check{\theta}}^{\tilde{\theta}} [\frac{d\tilde{U}(\theta,\hat{\chi}(\theta,\alpha(y)),\alpha(y),\beta(y))}{d\theta}\Big|_{\theta=y}]dy \geq \int_{\check{\theta}}^{\tilde{\theta}} \frac{d\tilde{U}(\theta,\hat{\chi}(\theta,\alpha(\check{\theta})),\alpha(\check{\theta}),\beta(\check{\theta}))}{d\theta}d\theta = \tilde{U}(\tilde{\theta},\hat{\chi}(\tilde{\theta},\alpha(\check{\theta})),\alpha(\check{\theta}),\beta(\check{\theta})) - \tilde{U}(\check{\theta},\hat{\chi}(\check{\theta},\alpha(\check{\theta})),\alpha(\check{\theta}),\beta(\check{\theta})) \tag{22}$$

and thus (20) is verified and hence, the IC constraint holds true. In a similar way, we can show that IC constraint holds true for $\tilde{\theta} < \check{\theta}$.

In the second part of the proof, we show that given an optimal solution to (7), we can find a solution to optimization (15) with the same value of the objective. As the first step, we prove that the IC constraint in (7) implies the monotonicity of $\alpha(\theta)$. Let's consider an agent of type $\tilde{\theta}$ with an announced type $\check{\theta} = \tilde{\theta} - \epsilon$, where $\epsilon > 0$ and then $\epsilon \to 0$, hence the IC constraint gives

$$(\alpha(\tilde{\theta}) - p)\hat{\chi}(\tilde{\theta},\alpha(\tilde{\theta})) + \beta(\tilde{\theta}) + \tilde{\theta}\pi(\hat{\chi}(\tilde{\theta},\alpha(\tilde{\theta}))) \geq (\alpha(\check{\theta}) - p)\hat{\chi}(\tilde{\theta},\alpha(\check{\theta})) + \beta(\check{\theta}) + \tilde{\theta}\pi(\hat{\chi}(\tilde{\theta},\alpha(\check{\theta})) \tag{23}$$

and if we consider an agent of type $\check{\theta}$ with the announced type equal to $\tilde{\theta}$, then the IC constraint reads as

$$(\alpha(\check{\theta}) - p)\hat{\chi}(\check{\theta},\alpha(\check{\theta})) + \beta(\check{\theta}) + \check{\theta}\pi(\hat{\chi}(\check{\theta},\alpha(\check{\theta}))) \geq (\alpha(\tilde{\theta}) - p)\hat{\chi}(\check{\theta},\alpha(\tilde{\theta})) + \beta(\tilde{\theta}) + \check{\theta}\pi(\hat{\chi}(\check{\theta},\alpha(\tilde{\theta}))). \tag{24}$$

By summation of (23) and (24) and rearranging the terms, we get

$$(\alpha(\tilde{\theta}) - p)\Big(\hat{\chi}(\tilde{\theta},\alpha(\tilde{\theta})) - \hat{\chi}(\check{\theta},\alpha(\tilde{\theta}))\Big) + \tilde{\theta}\Big(\pi(\hat{\chi}(\tilde{\theta},\alpha(\tilde{\theta}))) - \pi(\hat{\chi}(\tilde{\theta},\alpha(\check{\theta})))\Big) \geq (\alpha(\check{\theta}) - p)\Big(\hat{\chi}(\tilde{\theta},\alpha(\check{\theta})) - \hat{\chi}(\check{\theta},\alpha(\check{\theta}))\Big) + \check{\theta}\Big(\pi(\hat{\chi}(\check{\theta},\alpha(\tilde{\theta}))) - \pi(\hat{\chi}(\check{\theta},\alpha(\check{\theta})))\Big). \tag{25}$$

Dividing (25) by ϵ, we have

$$(\alpha(\tilde{\theta}) - p)(\frac{\partial\hat{\chi}}{\partial\theta})_{\tilde{\theta}} + \tilde{\theta}(\frac{\partial\pi}{\partial\hat{\theta}})_{\tilde{\theta}} \geq (\alpha(\check{\theta}) - p)(\frac{\partial\hat{\chi}}{\partial\theta})_{\tilde{\theta}} + \check{\theta}(\frac{\partial\pi}{\partial\hat{\theta}})_{\tilde{\theta}}. \tag{26}$$

Equation (26) can be rewritten as

$$(\alpha(\tilde{\theta}) - \alpha(\check{\theta}))(\frac{\partial\hat{\chi}}{\partial\theta})_{\tilde{\theta}} + (\tilde{\theta} - \check{\theta})(\frac{\partial\pi}{\partial x})_{\hat{\chi}}\frac{\partial\hat{\chi}}{\partial\alpha}(\frac{\partial\alpha}{\partial\hat{\theta}})_{\tilde{\theta}} \geq 0. \tag{27}$$

Dividing (27) again by ϵ, gives

$$(\frac{\partial\alpha}{\partial\hat{\theta}})_{\tilde{\theta}}\Bigg((\frac{\partial\hat{\chi}}{\partial\theta})_{\tilde{\theta}} + (\frac{\partial\pi}{\partial x})_{\hat{\chi}}\frac{\partial\hat{\chi}}{\partial\alpha}\Bigg) \geq 0. \tag{28}$$

Since $\frac{\partial\pi}{\partial x}, \frac{\partial\hat{\chi}}{\partial\alpha}$ and $\frac{\partial\hat{\chi}}{\partial\theta}$ are positive as the results of Assumption 1, Remark 2, and the fact that $\tilde{\theta}$ is any arbitrary point in Θ, we can conclude that $\dot{\alpha}(\hat{\theta}) = \frac{\partial\alpha}{\partial\hat{\theta}} \geq 0$.

To derive constraint (15c), we rearrange (23) and (24) as follows

$$(\alpha(\check{\theta}) - p)\hat{\chi}(\tilde{\theta},\alpha(\check{\theta})) + \tilde{\theta}\pi(\hat{\chi}(\tilde{\theta},\alpha(\check{\theta}))) - (\alpha(\tilde{\theta}) - p)\hat{\chi}(\tilde{\theta},\alpha(\tilde{\theta})) - \tilde{\theta}\pi(\hat{\chi}(\tilde{\theta},\alpha(\tilde{\theta}))) \leq \beta(\tilde{\theta}) - \beta(\check{\theta}) \leq (\alpha(\check{\theta}) - p)\hat{\chi}(\check{\theta},\alpha(\check{\theta})) + \check{\theta}\pi(\hat{\chi}(\check{\theta},\alpha(\check{\theta}))) - (\alpha(\tilde{\theta}) - p)\hat{\chi}(\check{\theta},\alpha(\tilde{\theta})) - \check{\theta}\pi(\hat{\chi}(\check{\theta},\alpha(\tilde{\theta}))). \tag{29}$$

Dividing (29) by ϵ, we have

$$[\frac{d}{d\hat{\theta}}[(\alpha(\hat{\theta}) - p)\hat{\chi}(\theta,\alpha(\hat{\theta})) + \theta\pi(\hat{\chi}(\theta,\alpha(\hat{\theta})))]]\Big|_{\substack{\hat{\theta}=\tilde{\theta}\\ \theta=\tilde{\theta}}} \leq \frac{d}{d\tilde{\theta}}\beta(\tilde{\theta}) \leq [\frac{d}{d\hat{\theta}}[(\alpha(\hat{\theta}) - p)\hat{\chi}(\theta,\alpha(\hat{\theta})) + \theta\pi(\hat{\chi}(\theta,\alpha(\hat{\theta})))]]\Big|_{\substack{\hat{\theta}=\tilde{\theta}\\ \theta=\tilde{\theta}}} \tag{30}$$

which implies that

$$\frac{d}{d\tilde{\theta}}\beta(\tilde{\theta}) = [\frac{d}{d\hat{\theta}}[(\alpha(\hat{\theta}) - p)\hat{\chi}(\theta,\alpha(\hat{\theta})) + \theta\pi(\hat{\chi}(\theta,\alpha(\hat{\theta})))]]\Big|_{\substack{\hat{\theta}=\tilde{\theta}\\ \theta=\tilde{\theta}}}. \tag{31}$$

Integrating (31) with respect to $\tilde{\theta}$ from $\underline{\theta}$ to $\tilde{\theta}$, we have

$$\beta(\tilde{\theta}) - \beta(\underline{\theta}) = \int_{\underline{\theta}}^{\tilde{\theta}} [\frac{d}{d\hat{\theta}}[(\alpha(\hat{\theta}) - p)\hat{\chi}(\theta,\alpha(\hat{\theta})) + \theta\pi(\hat{\chi}(\theta,\alpha(\hat{\theta})))]]\Big|_{\substack{\hat{\theta}=\tilde{\theta}\\ \theta=\tilde{\theta}}} d\tilde{\theta}. \tag{32}$$

Considering $U(\underline{\theta},\underline{\theta}) = 0$ from Proposition 2 and the integration by parts, (32) can be rewritten as follows

$$\beta(\tilde{\theta}) = \int_{\underline{\theta}}^{\tilde{\theta}} \Big[\frac{d}{d\theta}[(\alpha(\hat{\theta}) - p)\hat{\chi}(\theta,\alpha(\hat{\theta})) + \theta\pi(\hat{\chi}(\theta,\alpha(\hat{\theta})))]\Big]\Big|_{\hat{\theta}=\theta} d\theta - [(\alpha(\tilde{\theta}) - p)\hat{\chi}(\tilde{\theta},\alpha(\tilde{\theta})) + \tilde{\theta}\pi(\hat{\chi}(\tilde{\theta},\alpha(\tilde{\theta})))]. \tag{33}$$

Considering the definition of K, equation (33) can be rewritten as follows which completes the proof.

$$\beta(\tilde{\theta}) = \int_{\underline{\theta}}^{\tilde{\theta}} K_\theta(\hat{\chi}(\theta,\alpha(\hat{\theta})),\theta,\alpha(\hat{\theta}))\Big|_{\hat{\theta}=\theta} d\theta - K(\hat{\chi}(\tilde{\theta},\alpha(\tilde{\theta})),\tilde{\theta},\alpha(\tilde{\theta})). \tag{34}$$

■

Substituting $\beta(\tilde{\theta})$ from (34) into the cost function of (15) results in an optimization with double integrals. In order to simplify and solve this optimization problem, we make the following assumption.

Definition 3: As is customary in the literature [21], $h(\theta) \equiv f(\theta)/(1-F(\theta))$ denotes as the hazard rate of type θ. Here, $F(\theta)$ and $f(\theta)$ are the cumulative distribution function and probability density function of the random variable θ, respectively.

Assumption 2: For each $\theta \in \Theta$, $h(\theta)$ is increasing [21].

Proposition 3: Optimization (15) is equivalent to

$$\max_{\alpha(\hat{\theta})} \int_{\underline{\theta}}^{\bar{\theta}} \Big[g(\hat{\chi}(\hat{\theta},\alpha(\hat{\theta}))) - [K_\theta(\hat{\chi}(\theta,\alpha(\hat{\theta})),\theta,\alpha(\hat{\theta}))]\Big|_{\theta=\hat{\theta}} \frac{1}{h(\hat{\theta})} \tag{35a}$$

$$+ \hat{\theta}\pi(\hat{\chi}(\hat{\theta},\alpha(\hat{\theta}))) - p\hat{\chi}(\hat{\theta},\alpha(\hat{\theta}))\Big] f(\hat{\theta})d\hat{\theta}$$

$$s.t.\ \dot{\alpha}(\hat{\theta}) \geq 0 \tag{35b}$$

$$\hat{\chi}(\theta,\alpha(\hat{\theta})) = \Gamma(\frac{p-\alpha(\hat{\theta})}{\theta}). \tag{35c}$$

Proof: Replacing $\beta(\hat{\theta})$ from (15c) in the utility function of task publisher, we have

$$\mathbb{E}_\theta[V] = \int_{\underline{\theta}}^{\bar{\theta}} \Big[g(\hat{\chi}(\hat{\theta},\alpha(\hat{\theta}))) - \alpha(\hat{\theta})\hat{\chi}(\hat{\theta},\alpha(\hat{\theta})) \tag{36}$$

$$- \int_{\underline{\theta}}^{\hat{\theta}} [K_\theta(\hat{\chi}(\theta,\alpha(\hat{\theta})),\theta,\alpha(\hat{\theta}))]\Big|_{\hat{\theta}=\theta} d\theta$$

$$+ K(\hat{\chi}(\theta,\alpha(\hat{\theta})),\theta,\alpha(\hat{\theta}))\Big|_{\theta=\hat{\theta}}) \Big] f(\hat{\theta})d\hat{\theta}.$$

The integration by parts of the term $\int_{\underline{\theta}}^{\bar{\theta}} \Big[\int_{\underline{\theta}}^{\tilde{\theta}} K_\theta(X(\theta,\alpha(\hat{\theta})),\theta,\alpha(\hat{\theta}))\Big|_{\hat{\theta}=\theta} d\theta \Big] f(\tilde{\theta})d\tilde{\theta}$ gives

$$\int_{\underline{\theta}}^{\bar{\theta}} \Big[\int_{\underline{\theta}}^{\tilde{\theta}} K_\theta(\hat{\chi}(\theta,\alpha(\hat{\theta})),\theta,\alpha(\hat{\theta}))\Big|_{\hat{\theta}=\theta} d\theta \Big] f(\tilde{\theta})d\tilde{\theta} = \tag{37}$$
$$\int_{\underline{\theta}}^{\bar{\theta}} [K_\theta(\hat{\chi}(\theta,\alpha(\hat{\theta})),\theta,\alpha(\hat{\theta}))]\Big|_{\theta=\hat{\theta}} \frac{1-F(\tilde{\theta})}{f(\tilde{\theta})} f(\tilde{\theta})d\tilde{\theta}.$$

Considering the definition of $h(t)$ in Definition 3, (37) can be rewritten as cost function (35a) which completes the proof. ■

In what follows, we investigate the solution of optimization (35) by rewriting it as an optimal control problem. We also add the following assumption on the derivative of function α.

Assumption 3: There exists $\bar{u} > 0$ such that $\forall \hat{\theta} \in \Theta$, $\dot{\alpha}(\hat{\theta}) \leq \bar{u}$. Optimization (35) can be rewritten as the following optimal control problem with $\alpha(\hat{\theta})$ as the state and $u(\hat{\theta})$ as the control input.

$$\max_{\alpha(\hat{\theta})} \int_{\underline{\theta}}^{\bar{\theta}} V_{sp}(\hat{\theta},\alpha(\hat{\theta}))d\hat{\theta} \tag{38a}$$

$$s.t.\quad \dot{\alpha}(\hat{\theta}) = u(\hat{\theta}) \tag{38b}$$

$$u(\hat{\theta}): \Theta \to \mathbb{U} := [0,\bar{u}] \tag{38c}$$

$$\hat{\chi}(\theta,\alpha(\hat{\theta})) = \Gamma(\frac{p-\alpha(\hat{\theta})}{\theta}) \tag{38d}$$

where $V_{sp}(\hat{\theta},\alpha(\hat{\theta})) = \big[g(\hat{\chi}(\hat{\theta},\alpha(\hat{\theta}))) - [K_\theta(\hat{\chi}(\theta,\alpha(\hat{\theta})),\theta,\alpha(\hat{\theta}))]\big|_{\theta=\hat{\theta}} \frac{1}{h(\hat{\theta})} + \hat{\theta}\pi(\hat{\chi}(\hat{\theta},\alpha(\hat{\theta}))) - p\hat{\chi}(\hat{\theta},\alpha(\hat{\theta}))\big] f(\hat{\theta})$. To solve optimization problem (38), we substitute the variable $\hat{\chi}(\hat{\theta},\alpha(\hat{\theta}))$ within the cost function $V_{sp}(\hat{\theta},\alpha(\hat{\theta}))$ with the corresponding term derived from constraint (38d). In the next step, we introduce the Hamiltonian function as follows [22]:

$$H(\hat{\theta},\alpha(\hat{\theta}),u(\hat{\theta}),\lambda(\hat{\theta})) = V_{sp}(\hat{\theta},\alpha(\hat{\theta})) + \lambda(\hat{\theta})u(\hat{\theta}) \tag{39}$$

where λ is a Lagrange multiplier. Next, we present a proposition that provides both necessary and sufficient optimality conditions for optimization (38) and then, a numerical algorithm is proposed in Algorithm 1 to find the solution.

Proposition 4: The control and state functions $u(\hat{\theta})$, $\alpha(\hat{\theta})$ are the solution of the optimization (38) if and only if the following conditions which are known as Minimum Principle are met.

$$\dot{\alpha}(\hat{\theta}) = \frac{\partial}{\partial\lambda} H(\hat{\theta},\alpha(\hat{\theta}),u(\hat{\theta}),\lambda(\hat{\theta})) \tag{40}$$

$$\dot{\lambda}(\hat{\theta}) = \frac{\partial}{\partial\alpha} H(\hat{\theta},\alpha(\hat{\theta}),u(\hat{\theta}),\lambda(\hat{\theta})) \tag{41}$$

$$u(\hat{\theta}) = \arg\min_{u\in\mathbb{U}} H(\hat{\theta},\alpha(\hat{\theta}),u(\hat{\theta}),\lambda(\hat{\theta})) \tag{42}$$

$$\lambda(\bar{\theta}) = 0. \tag{43}$$

Proof: As shown in [22, chapter 3], since $\dot{\alpha}(\hat{\theta})$ is the linear function of $\alpha(\hat{\theta})$ and $u(\hat{\theta})$ and also $V_{sp}(\hat{\theta},\alpha(\hat{\theta}))$ is a concave function and $\mathbb{U}$ is a convex set, the conditions of the minimum principle are both necessary and sufficient for optimality. ■

Gradient Projection Algorithm [23] can be utilized to solve this optimal control problem as presented in Algorithm 1. Note that Algorithm 1 is designed such that both IC and IR constraints are met at each iteration, even if full convergence is not achieved.

Algorithm 1 The Gradient Projection Algorithm for solving optimal control problem that the task publisher is faced to design mechanism.

1: Choose a sequence $\{\gamma^k\}$ of positive real number, parameter ϵ as the stopping condition for the algorithm and an initial control function $u^0(\hat{\theta}): \Theta \to \mathbb{U}$. Also, set $i = 0$.
2: Using the control function $u^k(\hat{\theta})$, solve differential equation (40) with initial condition $\alpha(\underline{\theta}) = \alpha_0$ and calculate function $\alpha^k(\hat{\theta})$.
3: Using the control function $u^k(\hat{\theta})$ and state function $\alpha^k(\hat{\theta})$ from previous steps, solve differential equation (41) with initial condition $\lambda(\bar{\theta}) = 0$ and calculate function $\lambda^k(\hat{\theta})$.
4: Update control function by $u^{k+1}(\hat{\theta}) = P_U(u^k(\hat{\theta}) - \gamma^k \frac{\partial H}{\partial u^k})$.
5: If $||u^{k+1}(\hat{\theta}) - u^k(\hat{\theta})||_2 < \epsilon$ then stop. Otherwise, replace k with $k+1$ and go to step 2.

Proposition 5: If the sequence of learning rate $\{\gamma^k\}$ be chosen such that $\sum_{k=0}^{\infty}\gamma^k = \infty$, and $\lim_{k\to\infty}\gamma^k = 0$, then the sequence u^i in Algorithm. 1 converges to optimal control function.

Proof: The proof is presented in [23, Corollary 3.8]. ■

Proposition 6: In Algorithm 1, the convergence rate of the objective function in (38) to its optimal value is in the order of $O(\frac{1}{k})$.

Proof: The proof is presented in [24, Theorem 2.1.14]. ■

Remark 3: The Euler discretization method [23] is employed for the initialization of the control function as a piece-wise constant function represented by $u^0(\hat{\theta}) = u_i$ for $\hat{\theta} \in [\theta_i, \theta_{i+1})$, $i = 0,\dots,g-1$, where g is a positive integer, $\theta_0 < \theta_1 < \cdots < \theta_g$ and $u_0,\dots,u_{g-1}$ are arbitrary constant values belonging to the sets Θ and $\mathbb{U}$, respectively.

IV. ILLUSTRATIVE EXAMPLE

In this section, we address the realm of crowdsensing tasks pertaining to the collection of data from a group of individual sources, often facilitated by personal mobile devices and sensor-equipped tools [6], [17]. The functions $\pi(x) = \frac{z_2}{1-z_1}x^{1-z_1}$ and $g(x) = \frac{1}{1-q_1}x^{1-q_1}$ are used, with the values $q_1 = 0.5$, $z_1 = 0.5$, and $q_2 = 3$ as the revenue that agents and task publisher received when agents record the data by their sensors. We assume that the types of agents are uniformly distributed within the interval $[4,6]$ and the cost of recording and sending one unit of data is set at $p = 10$. The initial control function is set to $u^0(\hat{\theta}) = 0.2$ for $\hat{\theta} \in [4,4.5)$, $u^0(\hat{\theta}) = 0.3$ for $\hat{\theta} \in [4.4,5)$,$u^0(\hat{\theta}) = 0.5$ for $\hat{\theta} \in [5,5.5)$, and $u^0(\hat{\theta}) = 0.7$ for $\hat{\theta} \in [5.5,6]$. The learning rate is $\gamma^i = 0.01$. Using these functions and values, we can calculate $\hat{\chi}$ using (8) as $\hat{\chi}(\theta,\alpha(\hat{\theta})) = (\frac{p-\alpha(\hat{\theta})}{\theta})^{-\frac{1}{z_1}}$. The convergence of Algorithm 1 is depicted in Figure 2, which shows that $\alpha(\hat{\theta})$ converges after approximately 300 iterations.

To demonstrate the validity of the IR and IC constraints in the proposed scheme, the utilities of five specific agent types ($\theta = 4, 4.5, 5, 5.5, 6$) when they announce different types are shown in Figure3. The black stars on the curve represent the points at which each type of agent obtains maximum utility. Figure 3 illustrates that when an agent truthfully announces its type, it receives a positive and maximal utility. However, when the agent deceives the task publisher by announcing a false type, it incurs a loss. These results confirm that the proposed mechanism satisfies the IR and IC constraints.

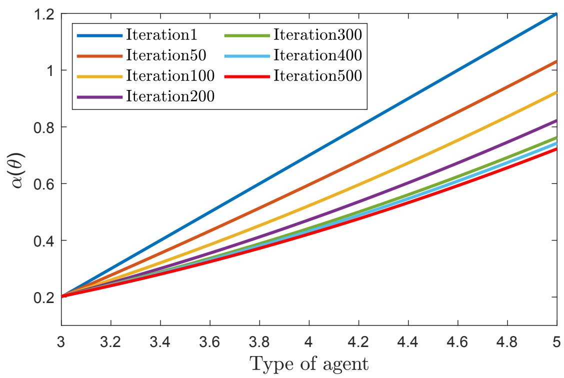

Fig. 2. $\alpha(\hat{\theta})$ allocated to different types of agents in different iterations.

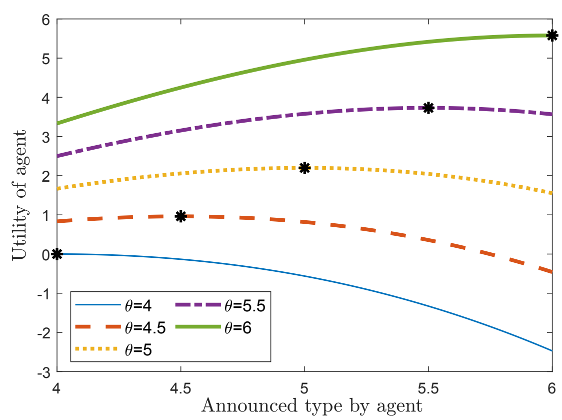

Fig. 3. Utilities of different agents when announcing different types.

V. Conclusion

This paper presented a new profit maximization mechanism for task allocation under incomplete information about the utilities of autonomous agents. The proposed mechanism allows the task publisher to maximize its utility and simultaneously, ensures both truthful reporting of the agents' private information and allows autonomous agents to decide on their own participation levels. We formulated the optimal truthful mechanism as a nonconvex functional optimization problem. By establishing a relation between the decision functions of the task publisher, we found an equivalent representation of the incentive constraint, which transformed the nonconvex optimization into a tractable convex optimal control problem.

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