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Generalized Stochastic Dynamic Aggregative Game for Demand-Side Management in Microgrids with Shared Battery

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***Abstract*—In this paper, we focus on modeling and analysis of demand-side management in a microgrid where agents utilize grid energy and a shared battery charged by renewable energy sources. We model the problem as a generalized stochastic dynamic aggregative game with chance constraints that capture the effects of uncertainties in the renewable generation and agents' demands. Computing the solution of the game is a complex task due to probabilistic and coupling constraints among the agents through the state of charge of the shared battery. We investigate the Nash equilibrium of this game under uncertainty considering both the uniqueness of the solution and the effect of uncertainty on the solution. Simulation results demonstrate that the presented stochastic method is superior to deterministic methods.**

***Index Terms*—Stochastic dynamic game, chance constraints, microgrids, shared battery, renewable energy sources.**

I. Introduction

MULTI-agent coordination for energy systems has emerged as a highly effective approach to achieve energy savings and maintain stability in microgrid systems. To accomplish this objective, concepts and techniques from game theory has been utilized due to their ability to incorporate user behavior and optimization perspectives [1].

In this paper, we study demand-side management (DSM) in microgrids as a *generalized stochastic dynamic aggregative game* with uncertainties in renewable energy and demand, using a shared battery and selfish residential consumers. Additionally, operational constraints were taken into account in modeling the problem. To avoid suffering from peak loads, a term related to the cumulative exchange of power between the agents and the grid appears in the cost function of the agents as part of electricity tariff. This leads to the aggregative form of the proposed game model. Unlike previous studies, a more comprehensive form of stochastic constraints is considered in the form of chance constraints. Due to the incorporation of a shared battery and accounting for the impact of uncertain sources, the state of charge (SoC) of the battery has stochastic dynamics shared between the agents. In addition, as we impose a constraint on the SoC of the shared battery, we have both dynamic and static stochastic coupling constraints among the agents, which shape the proposed game in a generalized, stochastic, and dynamic form. Then, we propose a series of reformulations and guaranteed under-approximations over the cost functions, stochastic dynamics, and chance constraints such that the game is converted into a static generalized aggregative form. Finally, we verify the conditions under which a Nash-seeking method can obtain the game's equilibrium point.

Related Works. Game theory analyzes strategic interactions in demand-side management, modeling consumer behavior and optimizing incentives for efficient resource use. The main types of games studied in this context are *dynamic games* that focus on the dynamic relationships among the agents (e.g., shared battery resources, dynamic pricing, etc.), and *generalized aggregative games*, which account for the coupling constraints among the agents. Comprehensive reviews on the analysis and decomposition of dynamic games can be found in [2], [3]. Various theoretical approaches have been employed to solve dynamic games utilizing optimal control theory to study open-loop and closed-loop Nash equilibrium [3]–[5]. For deterministic finite horizon discrete-time dynamic games, the state dynamic equation can be treated as a finite number of constraints which transforms the problem into a generalized aggregative game [6], however, increasing the number of constraints can impose high computational cost. Stochastic dynamic games have also been studied for both continuous-time [7], [8] and discrete-time systems [9], [10]. However, these works do not study chance constraints on the system's state or coupling constraint on the control inputs of agents. In contrast to previous studies that use almost-sure satisfaction of constraints [11], [12], we employ chance constraints due to their flexibility and potential for better outcomes with a predefined confidence. Imposing deterministic constraints on estimated random variables have also been considered in [13]–[15], but this approach overlooks the inherent stochastic nature of the uncertainties, increasing the risk of practical implementation failures. From an application perspective, DSM is addressed through dynamic games with [16], [17] or without [18], [19] the presence of shared battery. DSM has also been studied through generalized aggregative games [13], [20], [21]. In [13], [21], the impact of shared dynamics was not taken into account. These works studied the game in deterministic setups without considering uncertainty sources in microgrid systems. In this paper, we are dealing with a DSM game with stochastic shared dynamics of battery and coupling chance constraint on the state, referred to as a generalized stochastic

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dynamic aggregative game.

Contributions. The contributions of this paper are:

- Introducing a novel framework for demand-side management as a generalized stochastic dynamic aggregative game, featuring constraints in the form of stochastic dynamics and chance constraints.
- Providing an under-approximation for the feasible set of the stochastic dynamic game with chance constraints which results in a generalized static aggregative game.
- Analyzing the existence and uniqueness of the generalized Nash equilibrium (GNE) of the game, and proposing a semi-decentralized Nash seeking algorithm.

Notations. $\mathbb{R}$ denotes the set of real numbers. $\mathbf{0}_\tau(\mathbf{1}_\tau)$ denotes a vector with dimention $\tau \times 1$ that all elements equal to $0(1)$. $\mathbf{1}_{\tau\times\tau}$ denotes a matrix with dimention $\tau \times \tau$ that all elements equal to 1. $\mathbf{I}_\tau$ denotes a $\tau \times \tau$ identity matrix. $A \otimes B$ denotes the Kronecker product between matrices A and B. Suppose that we have N vectors, $\mathbf{x}_1, ..., \mathbf{x}_N \in \mathbb{R}^n$, then we define $\mathbf{x} \triangleq \left[\mathbf{x}_1^T, ..., \mathbf{x}_N^T\right]^T$. $\mathbf{M}_\tau$ is a $\tau \times \tau$ lower triangular matrix such that $M_\tau(i,j) = 1$ if $i \leq j$, and zero otherwise.

II. System Model

We investigate a grid-connected community microgrid as shown in Figure 1, which comprises N selfish residential households serving as agents in the system. Each household has the capacity to meet its own energy requirements via the power grid and a shared battery, which is recharged using renewable resources. Furthermore, we incorporate in our model the uncertainty in both the renewable energy sources and the energy demand. The microgrid operates under a tariff scheme with a retailer communicating the tariff information to the households. We consider the system where agents interact solely with a coordinator. All the decisions are taken over the day-ahead horizon.

A. Shared Battery Model

Battery plays an important role in microgrid systems by providing energy storage solutions that can balance energy supply and demand, and contribute in peak load shaving by proving a grid-free source of energy in peak hours. We consider that the battery is charged only through renewable energy and discharged by consumption of the agents from the battery. The state of charge (SoC) of the shared battery x^t is considered to have the following dynamics

$$x^{t+1} = x^t + \eta\Delta t\left[r^t - \sum_{j=1}^{N} u_j^t\right]. \tag{1}$$

At time t, u_j^t is the discharging decision of the battery by the j^{th} agent and r^t is the power generated by renewable energy sources, which is unknown and stochastic. The parameter η is the charging/discharging efficiency of the shared battery, and Δt is the sampling time.

Due to the stochastic input in (1), we consider the following chance constraints on the SoC and its final value:

$$\mathbf{Pr}\{\underline{x} \leq x^t \leq \bar{x}\} \geq 1 - \delta_x^t \quad \forall t \in \{0, \ldots, \tau\}, \tag{2}$$
$$\mathbf{Pr}\{|x^\tau - r| \leq \epsilon\} \geq 1 - \delta_x^{\tau,\text{final}}, \tag{3}$$

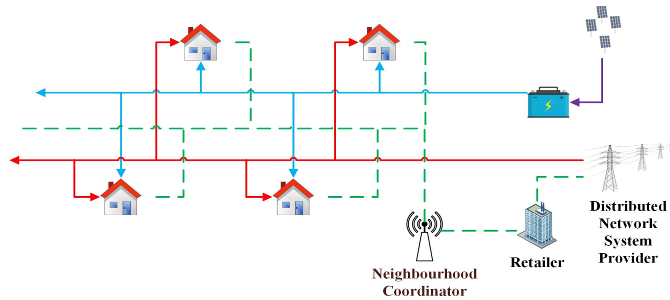

Fig. 1. System model. Red, blue and purple solid lines show respectively the energy supplied by the *Distributed Network Provider*, *shared battery* and *renewable energy*. Green dashed line shows *Essential Information Exchange (e.g. demands, common tariff).*

where $\underline{x}, \bar{x}, \delta_x^t$, and $\delta_x^{\tau,\text{final}}$ are positive constants in $[0,1]$ such that $\underline{x} < \bar{x}$, $r \in [\underline{x}, \bar{x}]$ is a positive constant, and $\epsilon \in (0, \min\{r - \underline{x}, \bar{x} - r\})$. The chance constraint in (3) guarantees that the SoC at the final time step lies within a specific range around r with a certain confidence. We also have the following constraint:

$$0 \leq u_i^t \leq \bar{u}_i \quad \forall t \in \{0, \ldots, \tau - 1\}. \tag{4}$$

B. Power exchange model

The load balance equation for household i at time $t \in \{0, ..., \tau - 1\}$ can be written as $g_i^t = d_i^t - u_i^t$, where g_i^t is power exchange of the i^{th} agent with the grid and d_i^t is its stochastic power demand. Let $g^t \triangleq \sum_{i=1}^N g_i^t$. The retailer imposes the following constraints on the allowable power exchange with the community of the microgrid:

$$\mathbf{Pr}\left\{0 \leq g^t \leq \bar{g}\right\} \geq 1 - \delta_g^t \quad \forall t \in \{0, \ldots, \tau - 1\}, \tag{5}$$

where $\bar{g}$ is the maximum power supply of the retailer, and $\delta_g^t \in [0, 1]$.

C. Electricity tarrifs

All the agents in the neighbourhood are billed using common electricity tariffs, modelled as

$$\pi(g^t) = K_{ToU}^t + \sum_{j=1}^{N} g_j^t k_c^N , \tag{6}$$

where $\pi(g^t)$ is the common electricity tariff for agents at time t and K_{ToU}^t is the conventional time-of-use pricing tariff that could depend on hour of the day. The positive constant k_c^N influences the cost in proportion to the peak power consumption creates a balance between the microgrid's objectives and ensuring fairness, peak shaving, and stability in the tariff system [22]. As mentioned in [11], [22], k_c^N is selected to be inversely proportional to the number of agents (this dependency is denoted by the superscript N), which performs normalization in aggregative term of grid electricity use of agents. The tariff function (6) is based on this fact that increasing the price of electricity at times of peak demand will motivate a rational household (agent) to schedule the shared battery such that the community peak can be shaved [23], [24].

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D. Cost Function of Each Agent in Model

The cost function of each agent (household) is

$$J_i = \mathbf{E}\left\{ \sum_{t=0}^{\tau-1} \left[\pi\left(g^t\right) g_i^t + \sum_{j=1}^{N} \left(\alpha^{dch} \left(u_j^t\right)^2 + \beta^{dch} u_j^t \right) \right] \right\}, \tag{7}$$

where $\alpha^{dch}, \beta^{dch}$ are positive constants. The cost function has a conventional form used frequently in the literature (see e.g., [11], [13]). It consists of two parts: one related to the cost of electricity and the other to battery degradation, which serves as a proxy for the shared battery lifespan.

Let $\mathbf{u}_i = [u_i^0, u_i^1, \ldots, u_i^{\tau-1}]^T$. We are now equipped to delineate the game-theoretic setup for our demand-side management model, as

$$\mathcal{G} = \begin{cases} \textbf{Players: } \text{A set of residential agents } \mathcal{N} = \{1, 2, \ldots, N\} \\ \textbf{Strategies of Agents: } \mathbf{u}_i \qquad i \in \mathcal{N} \\ \textbf{Cost Functions: } J_i\left(\mathbf{u}_i, \mathbf{u}_{-i}\right) \qquad i \in \mathcal{N} \\ \textbf{Stochastic Dynamic: } (1) \\ \textbf{Constraints: } \begin{cases} \textbf{Local: } (4) \\ \textbf{Coupling (Chance Constraints): } (2), (3), (5) \end{cases} \end{cases}$$

As demonstrated above, $\mathcal{G}$ evidently corresponds to a generalized stochastic dynamic aggregative game.

III. Approximation with Generalized Static Aggregative Game

We raise the following assumption for constructing an under-approximation of the feasible set of the game.

***Assumption** 1:* The initial state x^0 is known and $\underline{x} \leq x^0 \leq \bar{x}$. Additionally, r^t and d_i^t are random variables with bounded support $[a^t, b^t]$ and $[c_i^t, f_i^t]$, and their mean values are available to each agent. Random variables r^t and d_i^t could in general be dependent with known dependency graph.

The above assumption is based on the fact that the power generated by renewable energy sources and power demand of agents are bounded. Let us denote the input constraints in (4) in vector format as $\mathbf{0}_\tau \leq \mathbf{u}_i \leq \bar{u}_i \mathbf{1}_\tau$.

***Proposition** 1:* The chance constraint (2) can be under-approximated with the deterministic inequalities

$$\left(\underline{x} - x^0\right) \mathbf{1}_\tau - \rho \mathbf{M}_\tau \mu_r + \mathbf{q}_{x,1} + \sum_{j=1}^{N} \rho \mathbf{M}_\tau \mathbf{u}_j \leq 0, \tag{8}$$

$$\left(x^0 - \bar{x}\right) \mathbf{1}_\tau + \rho \mathbf{M}_\tau \mu_r + \mathbf{q}_{x,2} - \sum_{j=1}^{N} \rho \mathbf{M}_\tau \mathbf{u}_j \leq 0, \tag{9}$$

where $\mu_r = \left[\mathbf{E}\left\{r^0\right\}, \mathbf{E}\left\{r^1\right\}, \ldots, \mathbf{E}\left\{r^{\tau-1}\right\}\right]^T$, $\mathbf{q}_{x,1} = \left[q_{x,1}^1, q_{x,1}^2, \ldots, q_{x,1}^\tau\right]^T$, $\mathbf{q}_{x,2} = \left[q_{x,2}^1, q_{x,2}^2, \ldots, q_{x,2}^\tau\right]^T$, with $q_{x,1}^t \triangleq \sqrt{-\rho^2 \nu_r^t \sum_{k=0}^{t-1} \left(b^k - a^k\right)^2 \mathrm{Ln}(\tilde{\delta}_x^t)}$, $q_{x,2}^t \triangleq \sqrt{-\rho^2 \nu_r^t \sum_{k=0}^{t-1} \left(b^k - a^k\right)^2 \mathrm{Ln}(\tilde{\delta}_x^t - \delta_x^t)}$, $0 \leq \tilde{\delta}_x^t \leq 1$, $0 \leq \delta_x^t - \tilde{\delta}_x^t \leq 1$, and $\rho \triangleq \eta \Delta t$. Moreover, $\nu_r^t = \chi(\hat{G}_r^t)/2$ with $\hat{G}_r^t$ being the undirected dependency graph of the random variables $r^0, \ldots, r^{t-1}$, and $\chi(\hat{G}_r^t)$ represents the chromatic number of this graph (see [25]).

Proof: The proof is based on deriving from (1) the explicit form of x^t as

$$x^t = x^0 + \sum_{k=0}^{t-1} \left\{ -\rho \left(\sum_{j=1}^{N} u_j^k \right) + \rho r^k \right\} \tag{10}$$

and using *Chernoff-Hoeffding inequality* [25]. More details about the proof can be found in [26]. ∎

***Proposition** 2:* The chance constraint (3) can be under-approximated with the deterministic inequalities

$$r - \epsilon - x^0 - \rho \mathbf{1}_\tau^T \mu_r + q_{x,\text{final},1} + \sum_{j=1}^{N} \rho \mathbf{1}_\tau^T \mathbf{u}_j \leq 0, \tag{11}$$

$$x^0 - r - \epsilon + \rho \mathbf{1}_\tau^T \mu_r + q_{x,\text{final},2} + \sum_{j=1}^{N} \left(-\rho \mathbf{1}_\tau^T\right) \mathbf{u}_j \leq 0, \tag{12}$$

where $q_{x,\text{final},1} = \sqrt{-\rho^2 \nu_r^\tau \sum_{k=0}^{\tau-1} \left(b^k - a^k\right)^2 \mathrm{Ln}(\tilde{\delta}_x^{\tau,\text{final}})}$, $q_{x,\text{final},2} = \sqrt{-\rho^2 \nu_r^\tau \sum_{k=0}^{\tau-1} \left(b^k - a^k\right)^2 \mathrm{Ln}(\delta_x^{\tau,\text{final}} - \tilde{\delta}_x^{\tau,\text{final}})}$, with $0 \leq \tilde{\delta}_x^{\tau,\text{final}} \leq 1$, and $0 \leq \delta_x^{\tau,\text{final}} - \tilde{\delta}_x^{\tau,\text{final}} \leq 1$.

***Proposition** 3:* The chance constraint (5) can be under-approximated with the deterministic inequalities

$$-\sum_{j=1}^{N} \mu_{d_j} + \mathbf{q}_{g,1} + \sum_{j=1}^{N} \mathbf{u}_j \leq 0, \tag{13}$$

$$-\bar{g} \mathbf{1}_\tau + \sum_{j=1}^{N} \mu_{d_j} + \mathbf{q}_{g,2} - \sum_{j=1}^{N} \mathbf{u}_j \leq 0, \tag{14}$$

where $\mu_{d_j} = \left[\mathbf{E}\left\{d_j^0\right\}, \mathbf{E}\left\{d_j^1\right\}, \ldots, \mathbf{E}\left\{d_j^{\tau-1}\right\}\right]^T$, $\mathbf{q}_{g,1} = \left[q_{g,1}^0, q_{g,1}^1, \ldots, q_{g,1}^{\tau-1}\right]^T$, $\mathbf{q}_{g,2} = \left[q_{g,2}^0, q_{g,2}^1, \ldots, q_{g,2}^{\tau-1}\right]^T$, with $q_{g,1}^t = \sqrt{-\nu_d^t \sum_{j=1}^{N} \left(f_j^t - c_j^t\right)^2 \mathrm{Ln}(\tilde{\delta}_g^t)}$, $q_{g,2}^t = \sqrt{-\nu_d^t \sum_{j=1}^{N} \left(f_j^t - c_j^t\right)^2 \mathrm{Ln}(\delta_g^t - \tilde{\delta}_g^t)}$, $0 \leq \tilde{\delta}_g^t \leq 1$, and $0 \leq \delta_g^t - \tilde{\delta}_g^t \leq 1$. Moreover, $\nu_d^t = \chi(\hat{G}_d^t)/2$ with $\hat{G}_d^t$ being the undirected dependency graph of the random variables $d_1^t, \ldots, d_N^t$.

The proofs of the above two propositions follow similar steps as that of Proposition 1 (more details in [26]). All the inequalities in (8)–(9) and (11)–(14) can be written as

$$\sum_{j=1}^{N} A \mathbf{u}_j \leq b, \tag{15}$$

with appropriately defined matrices A and b, and with explicit local constraint or local decision set

$$\Omega_i = \left\{\mathbf{u}_i \in \mathbb{R}^T | \mathbf{0}_\tau \leq \mathbf{u}_i \leq \bar{u}_i \mathbf{1}_\tau \right\}. \tag{16}$$

Reformulation of the cost function. We can rewrite the cost function (7) as

$$J_i = \mathbf{u}_i^T G \mathbf{u}_i + T_i \mathbf{u}_i + \left[\frac{1}{N} \sum_{j=1}^{N} \mathbf{u}_j^T \right] H \mathbf{u}_i + c_i \;, \tag{17}$$

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where

$$G = \alpha^{dch}\mathbf{I}_\tau,\ H = Nk_c^N\mathbf{I}_\tau,$$

$$T_i = \left(-K_{ToU} - k_c^N\mu_{d_i} - k_c^N\sum_{j=1}^{N}\mu_{d_j} + \beta^{dch}\mathbf{1}_\tau\right)^T$$

$$c_i = \mu_{d_i}^T K_{ToU} + \sum_{t=0}^{\tau-1}\left[k_c^N\mathbf{E}\left\{\sum_{j=1}^{N}d_i^t d_j^t\right\}\right.$$

$$\left. + \alpha^{dch}\sum_{j=1,j\neq i}^{N}\left(u_j^t\right)^2 + \left(-k_c^N\mu_{d_i^t} + \beta^{dch}\right)\sum_{j=1,j\neq i}^{N}u_j^t\right],$$

$$K_{ToU} = \left[K_{ToU}^0, K_{ToU}^1, \ldots, K_{ToU}^{\tau-1}\right]^T,$$

$$\mu_{d_i^t} = \mathbf{E}\left\{d_i^t\right\}, \mu_{d_j} = \left[\mathbf{E}\left\{d_j^0\right\}, \mathbf{E}\left\{d_j^1\right\}, \ldots, \mathbf{E}\left\{d_j^{\tau-1}\right\}\right]^T.$$

The above under-approximations and reformulation of the cost function gives the following game $\mathcal{G}'$:

$$\mathcal{G}' = \begin{cases} \textbf{Players: } \text{A set of residential agents } \mathcal{N} = \{1,2,\ldots,N\} \\ \textbf{Strategies of Agents: } \mathbf{u}_i \quad i\in\mathcal{N} \\ \textbf{Cost Functions: } J_i\left(\mathbf{u}_i,\mathbf{u}_{-i}\right) \quad i\in\mathcal{N} \\ \textbf{Constraints: } \begin{cases}\textbf{Local: } (16) \\ \textbf{Coupling (deterministic and static): } (15)\end{cases}\end{cases}$$

where $\mathbf{u}_{-i} = \mathrm{col}\left(\{\mathbf{u}_j\}_{j\neq i}\right)$. Each agent seeks to minimize its cost function, $J_i(\mathbf{u}_i,\mathbf{u}_{-i})$, while taking into account both the coupling constraint (15) and the local constraint (16).

IV. Game Theoretical Analysis

Utilizing the coupling constraint and local decision set, we define the collective feasible set $\mathcal{U}$ as

$$\mathcal{U} = \Omega \cap \left\{(\mathbf{y}_1,\ldots,\mathbf{y}_N)\in\mathbb{R}^{\tau N} | \sum_{i=1}^{N}A\mathbf{y}_i - b \leq 0\right\}, \quad (18)$$

where $\Omega = \Omega_1\times\Omega_2\times\cdots\times\Omega_N$. Additionally, we define the feasible decision set of agent $i\in\{1,\ldots,N\}$ as

$$\mathcal{U}_i(\mathbf{u}_{-i}) = \left\{\mathbf{y}_i\in\Omega_i | A\mathbf{y}_i \leq b - \sum_{j=1,j\neq i}^{N}A\mathbf{u}_{-i}\right\}.$$

Our local optimization problem can be formalized as a game theory setup:

$$\begin{cases}\min_{\mathbf{u}_i\in\Omega_i} J(\mathbf{u}_i,\mathbf{u}_{-i}) \\ \text{s.t.} \quad A\mathbf{u}_i \leq b - \sum_{j=1,j\neq i}^{N}A\mathbf{u}_j\end{cases} \quad \text{for all } i\in\{1,\ldots,N\} \quad (19)$$

In this context, we use the notion of Generalized Nash instead of Nash, since the coupling among the agents is not only in their cost functions but also in their constraints.

***Definition** 1 (Generalized Nash Equlibrium):* The collective strategy $\mathbf{u}^*$ is a generalized Nash equilibrium (GNE) of the game in (19) if $\mathbf{u}^*\in\mathcal{U}$ and for all $i\in\{1,\ldots,N\}$

$$J_i\left(\mathbf{u}_i^*,\mathbf{u}_{-i}^*\right) \leq \inf\left\{J_i(\mathbf{y},\mathbf{u}_{-i}^*) | \mathbf{y}\in\mathcal{U}_i(\mathbf{u}_{-i}^*)\right\}.$$

***Remark** 1:* In our study, we can easily demonstrate that for each $i\in\{1,\ldots,N\}$ and $\mathbf{y}\in\mathcal{U}_{-i}$, the local cost function $J_i(\cdot,\mathbf{y})$ is strictly convex (since G is positive definite) and continuously differentiable. Additionally, for each agent, its local decision set is non-empty, compact (i.e., closed and bounded), and convex (since it is a hypercube, see (16)).

We also consider the following relax assumption.

***Assumption** 2:* The collective feasible set satisfies Slater's constraint qualification.

***Proposition** 4:* Under Assumption 2, the GNE exists in our proposed game setup.

Proof: Based on Remark 1, Assumption 2 and by utilizing Brouwer's fixed point theorem [27, Proposition 12.7], it is evident that a GNE exists. ■

Our study aims to identify a subset of the GNE that exhibits favorable properties such as economic fairness and enhanced social stability. Typically, this subset can be determined by solving an appropriate variational inequality, known as a generalized variational inequality (GVI). However, since the local cost functions are continuously differentiable, we utilize a specific type of GVI from [28]. Define the pseudo-gradient $F:\mathcal{U}\to\mathbb{R}^{\tau N}$ as

$$F(\mathbf{u}) = \mathrm{col}\left(\{\partial_{\mathbf{u}_i}J(\mathbf{u}_i,\mathbf{u}_{-i})\}_{i\in\{1,\ldots,N\}}\right),$$

which is single-valued with $F(\mathbf{u}) = \Gamma\mathbf{u} + \Lambda$, where

$$\Gamma = \left(2\alpha^{dch} + k_c^N\right)\left(\mathbf{I}_{N\times N}\otimes\mathbf{I}_\tau\right) + k_c^N\left(\mathbf{1}_{N\times N}\otimes\mathbf{I}_\tau\right)$$

$$\Lambda = \begin{bmatrix}T_1 & T_2 & \ldots & T_N\end{bmatrix}^T.$$

Based on Remark 1 and [27, Proposition 12.4], any solution of standard variational inequality problem VI$(\mathcal{U},F)$ is a generalized Nash equilibrium of (19), However, the converse is not necessarily true.

***Proposition** 5:* The variational GNE exists and is unique.

Proof: It has been demonstrated in [27, Proposition 12.11] that under all the system model properties stated in Remark 1 and Assumption 2, a sufficient condition for the existence (uniqueness) of a variational GNE in our game setup (19) is F being monotone (strictly monotone). The definition of monotonicity (strict monotonicity) implies that if Γ is positive semidefinite (positive definite), then F is monotone (strictly monotone). Since $\left(2\alpha^{dch}+k_c^N\right)\left(\mathbf{I}_{N\times N}\otimes\mathbf{I}_\tau\right)$ and $k_c^N\left(\mathbf{1}_{N\times N}\otimes\mathbf{I}_\tau\right)$ are two commuting square matrices, then based on [29, Theorem 1.3.12], the eigenvalues of Γ are either $(2\alpha^{dch}+k_c^N+NK_c^N)$ or $(2\alpha^{dch}+k_c^N)$, both strictly positive, so Γ is positive definite and F is also strictly monotone. ■

***Remark** 2:* F is ζ-strongly monotone with $\zeta \in \left(0, 2\alpha^{dch}+k_c^N\right)$. This can be demonstrated by utilizing the definition of strong monotonicity, in a manner analogous to Proposition 5.

***Remark** 3:* Based on Lipschitz definition and Gershgorin circle Theorem [29, Theorem 6.1.1], F is l_F-Lipschitz with $l_F > 2\alpha^{dch} + (N+1)\,k_c^N$.

As mentioned in [28], projected-gradient algorithms for generalized equilibrium seeking in aggregative games are preconditioned forward-backward methods. Based on this, we propose a semi-decentralized preconditioned forward-backward algorithm to solve variational inequality problem VI$(\mathcal{U},F)$, as presented in Algorithm 1. In the proposed algorithm, we define α_i as real values that belong to the interval $\left(0,\frac{2\zeta}{l_F^2}\right)$

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for all $i \in \{1,\dots,N\}$ and $\gamma \in (0,\gamma_{max})$. We set $\gamma_{max} \triangleq \frac{1}{\|\mathbf{1}_N^T \otimes A\|^2}\left[\frac{1}{\alpha_{i,max}} - \frac{1}{2\frac{\zeta}{l_F^2}}\right]$.

Algorithm 1: Preconditioned Forward Backward

Initialization: $k \leftarrow 1$, $\mathbf{u}_i^1 \leftarrow \mathbf{u}_i^{\text{init}}$, $\lambda^1 \leftarrow \lambda^{\text{init}}$

Repeat

Agents: $\forall i \in \{1,\dots,N\}$

$$\mathbf{u}_i^{k+1} \leftarrow \text{proj}_{\Omega_i}\left[\left(\mathbf{I}_{\tau\times\tau} - 2\alpha_i\left(G + \frac{H^T}{N}\right)\right)\mathbf{u}_i^k - \alpha_i A^T \lambda^k - \alpha_i T_i^T\right]$$

Coordinator:

$$\lambda^{k+1} \leftarrow \text{proj}_{\mathbb{R}^m_{\geq 0}}\left[\lambda^k + \gamma\left(2\ A\sum_{j=1}^N \mathbf{u}_j^{k+1} - A\sum_{j=1}^N \mathbf{u}_j^k - b\right)\right]$$

$k \leftarrow k+1$

Until for all agents $\|\mathbf{u}_i^{k+1} - \mathbf{u}_i^k\| \leq \epsilon_u^{\text{stop}}$, $\|\lambda^{k+1} - \lambda^k\| \leq \epsilon_\lambda^{\text{stop}}$

Moreover, we denote $\alpha_{i,min}$ and $\alpha_{i,max}$ as the minimum and maximum values of α_i across all $i \in \{1,\dots,N\}$ respectively, i.e., $\alpha_{i,min} \triangleq \min_{i\in\{1,\dots,N\}} \alpha_i$ and $\alpha_{i,max} \triangleq \max_{i\in\{1,\dots,N\}} \alpha_i$. In this context, the parameters l_F and ζ are determined according to Remark 2 and Remark 3, respectively.

The sequence $\left(\text{col}(\mathbf{u}^k,\lambda^k)\right)_{k=0}^{\infty}$ defined by Algorithm 1, with step sizes $\alpha_i \in \left(0, \frac{2\zeta}{l_F^2}\right)$, for all $i \in \{1,\dots,N\}$, and $\gamma \in (0,\gamma_{max})$, with $\gamma_{max} \triangleq \frac{1}{\|\mathbf{1}_N^T\otimes A\|^2}\left[\frac{1}{\alpha_{i,max}} - \frac{1}{2\frac{\zeta}{l_F^2}}\right]$, globally converges to variational GNE based on [28, Theorem 1]. More details about the algorithm can also be found in [26].

V. Simulation

In this section, we consider a microgrid system consisting of $N = 20$ identical residential users. The parameters of the cost functions in (7) are assigned as $\alpha^{dch} = 8$, $\beta^{dch} = 10$, and $k_c^N = 0.015$. Additionally, the conventional time-of-use pricing tariff is provided in Table I. The initial SoC is $x_0 = 0.5$ with minimum and maximum SoC of $\underline{x} = 0.1$ and $\bar{x} = 0.9$. The battery has a total energy capacity of $E = 20000$ units with efficiency $\eta = 1/E$. The upper bound on the discharging power for each user i is $\bar{u}_i = E/(N\Delta t)$. In the chance constraints (2), (3), and (5), we set $\delta_x^t = 0.8$, $\delta_x^{\tau,\text{final}} = 0.9$, $r = 0.6$, $\epsilon = 0.05$, $\bar{g} = 600$ and $\delta_g^t = 0.8$. The time horizon is $\tau = 24$ hours with time step $\Delta t = 1$ hour. We consider d_i^t and r^t to have a bounded support with 25% deviation around their mean value. In our configuration, we set $\tilde{\delta}_x^t = \tilde{\delta}_x^{\tau,\text{final}} = \tilde{\delta}_g^t = 0.05$ and $\nu_r^t = \nu_d^t = 1$. For Algorithm 1, $l_F = 16.31$, $\gamma = 5.33\times10^{-4}$, and $\zeta = 4.24$. Additionally, $\alpha = 10^{-4}\mathbf{diag}\{p\}\otimes(\mathbf{I}_{\tau\times\tau})$, where $p = [251, 311, 317, 172, 131, 70, 28, 152, 129, 74, 32, 121, 80, 223, 35, 71, 292, 33, 251, 183]$.

According to Proposition 5, the variational generalized Nash equilibrium is unique in our stochastic approach. We apply our stochastic method for DSM and compare its results with two deterministic methods. In these deterministic methods, we consider two worst-case scenarios for agent demand (lower bound and upper bound of demands) and use the mean value of renewable energy at each time as its deterministic value.

TABLE I
CONVENTIONAL TIME-OF-USE PRICING TARIFF.

Time(t)	0 – 4	5 – 14	15 – 16	17 – 21	22 – 24
Tariff(K_{ToU}^t)	29.45	30	29.5	30.5	29.5

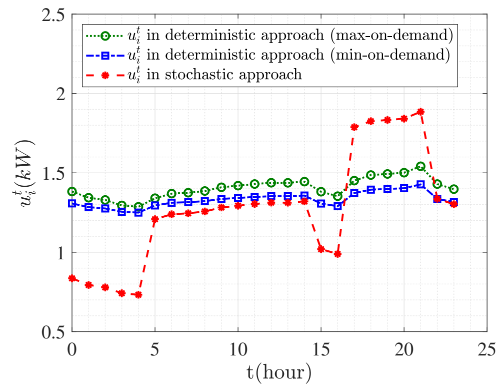

Fig. 2. Profile of $\boldsymbol{u_i^t}$ in stochastic and deterministic approaches.

The profile of u_i^t in the stochastic and deterministic approaches can be viewed in Figure 2. This figure illustrates that in periods when prices and demand are relatively low, the stochastic approach utilizes less battery energy compared to the deterministic methods. Conversely, during intervals with higher demand and electricity prices, the battery is employed to a greater extent under the stochastic method. Figure 3 illustrates the means of power exchange profile of the agents with the grid. The results in Figure 3 indicate that the stochastic approach of this paper performs peak shaving more effectively than the deterministic methods. Simulation results also reveal that, while the stochastic approach achieves more effective peak shaving than the deterministic methods, the power exchange profile of the agents with the grid increases more frequently compared to the deterministic approaches. We also compare, empirically, the cost function of our stochastic approach with the deterministic methods for 1,000 random demand values and display the histogram in Figure 4. As

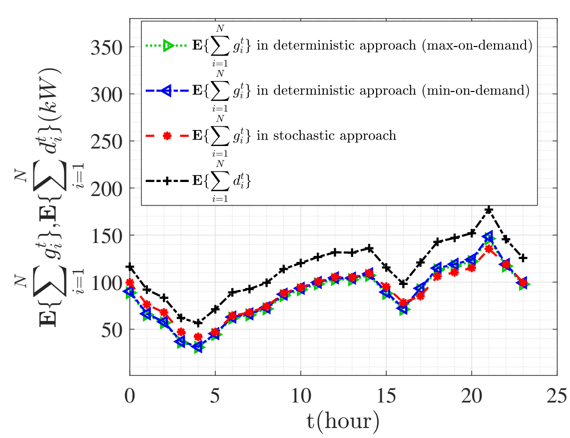

Fig. 3. Profile of power exchange of all the agents with the grid (aggregate demands of all the agents from the grid) in stochastic and deterministic approaches

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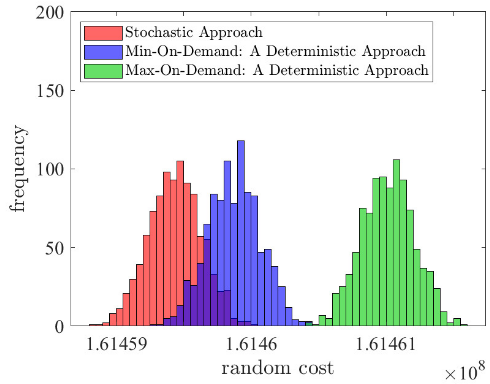

Fig. 4. Histogram comparing the random costs of stochastic and deterministic approaches.

evident from the figure, the expected cost for our stochastic approach is lower, showing that it incurs lower costs for agents.

VI. CONCLUSIONS

This paper studied a microgrid system where residential users use a shared battery charged through renewable sources and the grid. The model considered uncertain variables including user demand and renewable energy, as well as dynamic and stochastic coupling constraints. The paper used game theory to analyze the Nash equilibrium for demand-side management, examining existence and uniqueness. A semi-decentralized algorithm for Nash seeking was also proposed. The simulation results demonstrated the advantages of the proposed setup.

REFERENCES

[1] W. Saad, Z. Han, H. V. Poor, and T. Basar, "Game-theoretic methods for the smart grid: An overview of microgrid systems, demand-side management, and smart grid communications," *IEEE Signal Processing Magazine*, vol. 29, no. 5, pp. 86–105, 2012.
[2] T. Başar and G. Zaccour, *Handbook of Dynamic Game Theory*. Springer International Publishing AG, 2018.
[3] T. Başar and G. J. Olsder, *Dynamic noncooperative game theory*. SIAM, 1998.
[4] G. Jank and H. Abou-Kandil, "Existence and uniqueness of open-loop Nash equilibria in linear-quadratic discrete time games," *IEEE Transactions on Automatic Control*, vol. 48, no. 2, pp. 267–271, 2003.
[5] P. V. Reddy and G. Zaccour, "Open-loop Nash equilibria in a class of linear-quadratic difference games with constraints," *IEEE Transactions on Automatic Control*, vol. 60, no. 9, pp. 2559–2564, 2015.
[6] M. P. Abraham and A. A. Kulkarni, "New results on the existence of open loop Nash equilibria in discrete time dynamic games via generalized Nash games," *Mathematical Methods of Operations Research*, vol. 89, pp. 157–172, 2019.
[7] M. Huang, R. P. Malhamé, and P. E. Caines, "Large population stochastic dynamic games: closed-loop McKean-Vlasov systems and the Nash certainty equivalence principle," *Communications in Information and Systems*, vol. 6, no. 3, pp. 221 – 252, 2006.
[8] J. Zhao, Y. Guo, and Y. Wu, "On ε-Nash equilibria of linear time-delay dynamic games with convex strategy set," *Journal of the Franklin Institute*, vol. 359, no. 13, pp. 6567–6586, 2022.
[9] M. Pachter and K. Pham, "Discrete-time linear-quadratic dynamic games," *Journal of Optimization Theory and Applications*, vol. 146, pp. 151–179, 2010.
[10] F. G. Vélez, M. J. Lizárraga, and C. R. Carreon, "Open loop robust equilibria in uncertain discrete time games," *International Journal of Control, Automation and Systems*, vol. 19, pp. 587–595, 2021.
[11] H. Farzaneh, M. Shokri, H. Kebriaei, and F. Aminifar, "Robust energy management of residential nanogrids via decentralized mean field control," *IEEE Transactions on Sustainable Energy*, vol. 11, no. 3, pp. 1995–2002, 2020.
[12] S. Zou, J. Warrington, and J. Lygeros, "Game-theoretic robust energy coordination for a neighbourhood of smart homes," in *2019 18th European Control Conference (ECC)*, pp. 1–6, IEEE, 2019.
[13] A. Joshi, H. Kebriaei, V. Mariani, and L. Glielmo, "Decentralized control of residential energy storage system for community peak shaving: A constrained aggregative game," in *2021 IEEE Madrid PowerTech*, pp. 1–6, 2021.
[14] A.-H. Mohsenian-Rad, V. W. Wong, J. Jatskevich, R. Schober, and A. Leon-Garcia, "Autonomous demand-side management based on game-theoretic energy consumption scheduling for the future smart grid," *IEEE trans. on Smart Grid*, vol. 1, no. 3, pp. 320–331, 2010.
[15] C. P. Mediwaththe, E. R. Stephens, D. B. Smith, and A. Mahanti, "A dynamic game for electricity load management in neighborhood area networks," *IEEE Transactions on Smart Grid*, vol. 7, no. 3, pp. 1329–1336, 2015.
[16] C. P. Mediwaththe, E. R. Stephens, D. B. Smith, and A. Mahanti, "A dynamic game for electricity load management in neighborhood area networks," *IEEE Transactions on Smart Grid*, vol. 7, no. 3, pp. 1329–1336, 2016.
[17] C. P. Mediwaththe, M. Shaw, S. Halgamuge, D. B. Smith, and P. Scott, "An incentive-compatible energy trading framework for neighborhood area networks with shared energy storage," *IEEE Transactions on Sustainable Energy*, vol. 11, no. 1, pp. 467–476, 2019.
[18] M. Pilz and L. Al-Fagih, "A dynamic game approach for demand-side management: scheduling energy storage with forecasting errors," *Dynamic Games and Applications*, vol. 10, no. 4, pp. 897–929, 2020.
[19] R.-S. Liu, "An algorithmic game approach for demand side management in smart grid with distributed renewable power generation and storage," *Energies*, vol. 9, no. 8, p. 654, 2016.
[20] J. Jo and J. Park, "Demand-side management with shared energy storage system in smart grid," *IEEE Transactions on Smart Grid*, vol. 11, no. 5, pp. 4466–4476, 2020.
[21] R. Estrella, G. Belgioioso, and S. Grammatico, "A shrinking-horizon, game-theoretic algorithm for distributed energy generation and storage in the smart grid with wind forecasting," *IFAC-PapersOnLine*, vol. 52, no. 3, pp. 126–131, 2019.
[22] P. Chavali, P. Yang, and A. Nehorai, "A distributed algorithm of appliance scheduling for home energy management system," *IEEE Transactions on Smart Grid*, vol. 5, no. 1, pp. 282–290, 2014.
[23] S. Huang, Q. Wu, M. Shahidehpour, and Z. liu, "Dynamic power tariff for congestion management in distribution networks," *IEEE Transactions on Smart Grid*, vol. 10, no. 2, pp. 2148–2157, 2019.
[24] D. Steen, L. A. Tuan, and O. Carlson, "Effects of network tariffs on residential distribution systems and price-responsive customers under hourly electricity pricing," *IEEE Transactions on Smart Grid*, vol. 7, no. 2, pp. 617–626, 2016.
[25] S. Janson, "Large deviations for sums of partly dependent random variables," *Random Structures & Algorithms*, vol. 24, no. 3, pp. 234–248, 2004.
[26] S. Yadollahi, H. Kebriaei, and S. Soudjani, "Generalized stochastic dynamic aggregative game for demand-side management in microgrids with shared battery," *arXiv preprint arXiv:2310.02996*, 2023.
[27] D. P. Palomar and Y. C. Eldar, *Convex optimization in signal processing and communications*. Cambridge university press, 2010.
[28] G. Belgioioso and S. Grammatico, "Projected-gradient algorithms for generalized equilibrium seeking in aggregative games arepreconditioned forward-backward methods," in *2018 European Control Conference (ECC)*, pp. 2188–2193, IEEE, 2018.
[29] R. A. Horn and C. R. Johnson, *Matrix analysis*. Cambridge university press, 2012.