Age of Information in Scheduled Wireless Relay Networks

Masoumeh Moradian* and Aresh Dadlani†
*School of Computer Science, Institute for Research in Fundamental Sciences, Iran
†Department of Electrical and Computer Engineering, Nazarbayev University, Kazakhstan
Emails: mmoradian@ipm.ir, aresh.dadlani@nu.edu.kz

***Abstract*—In this paper, we use the concept of Markovian jump linear systems in order to analyze the age of information (AoI) in discrete-time Markovian systems. This approach is in fact, the discrete-time counterpart of the stochastic hybrid system (SHS) model reported for analyzing AoI in continuous-time Markovian systems and thus, is referred to as the discrete-time SHS (DT-SHS) model. We then apply the DT-SHS model in two wireless relay network settings to analyze and optimize the AoI associated with the static link scheduling policies. The first relay network comprises of one relay and a direct link between source and destination, whereas the second setting has two relays and no direct link. Moreover, a static link scheduling policy schedules the links without any knowledge about the state of the network. Using results obtained through numerical simulations, we validate our analytical approach and show the effect of relays in AoI improvement.**

***Index Terms*—Age of information, wireless relay network, discrete-time Markov chain, optimization, link scheduling.**

I. Introduction

In emerging communication networks such as real-time Internet of Things and machine-type communications, the end-nodes transmit status updates to a monitoring entity, which is responsible for making proper controlling decisions based on the received information. The timeliness of the received information in such networks is of crucial importance to ensure seemless operation of applications [1]. In this regard, quantifying the freshness of information, termed as *age of information* (AoI), has attracted a lot of attention in recent years [2]. AoI at a monitor is defined as the time elapsed since the most recent update generated at the monitor. Hence, defining $u(t)$ as the time stamp, i.e. the generation time of the most recent update, AoI[1] at time t can be characterized as $x(t) = t - u(t)$.

Conventionally, link scheduling problems are accounted for when optimizing the throughput or minimizing the average delay incurred by networks. Considering the fact that the AoI metric does not necessarily coincide with throughput and average delay [3], it is of interest to contemplate age-minimizing link scheduling policies in multi-hop networks. In particular, multiple relays may have common updates, i.e. updates from the same source, with different ages, while enduring different channel conditions in relay networks. Evidently, how a link scheduling policy should arrange the link transmissions under such conditions so as to optimize the AoI remains an open problem that demands scrutiny.

[1]Henceforth, the terms 'AoI' and 'age' are used interchangeably.

A number of studies related to age-optimal link scheduling in single-hop multi-source scenarios exist in the literature [4]–[8]. Particularly, the authors in [4] and [5] mainly focus on optimal link scheduling policies in broadcast channels. The optimal link scheduling policy reported in [4] is shown to be a stationary switch-type one, where the information of multiple sources are transmitted to their corresponding users through a base station (BS) by serving only one user at a time. In [5], the proposed optimal link scheduling policies minimize the expected weighted sum of the AoI of clients under unreliable BS-to-user channel conditions. The authors in [6]–[8] investigate the age-optimal link scheduling policies in multi-access scenarios. In [6], the authors propose a max-age matching policy for assigning the K orthogonal channels to N sensors in each time slot $(N > K)$ in order to minimize a symmetric and non-decreasing function of sensor ages. In [7], the authors schedule a shared channel among multiple users, where each user has a specific number of updates to be delivered. In [8], the stochastic random arrivals and user-side buffer management are incorporated into the link scheduling problem. All the aforementioned efforts however, apply to communication in single hop networks. Moreover, the updates transmitted or received by users do not share any commonality. That is to say, users receive or transmit the updates of distinct sources.

The AoI in two-hop networks with common information at the nodes has also been widely investigated [9]–[11]. The transmission times of an energy harvesting source and a half-duplex relay in a two-hop network are scheduled in a finite horizon to minimize the average AoI at the destination node in [9]. In [10], a source transmits updates through n relays to n^2 users, where the update transmissions experience i.i.d. random delays. The authors show that the earliest k transmission scheduling makes the AoI at the end users independent of n and then, they optimize the parameter k. In [11], N sensors that observe the updates generated by K sources at discrete times are scheduled to transmit their updates to a BS via an error-free shared link. Each sensor observes the update of each source with a specific probability. The age-optimal dynamic scheduling policies are designed with and without knowing the network parameters.

In this paper, we investigate the age-optimal static link scheduling policies in two different wireless relay networks where the static policy schedules the links without prior information on the network state. We particularly focus on two wireless relay networks; the first network setting consists

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of one relay and a direct link between source and destination. The second setting however, involves two relays and no direct link. The source and relay nodes in our proposed networks have common updates, i.e. the updates of the same source. While one node may have fresher updates, it may also have a worse channel condition. Thus, we investigate how the link scheduling policy schedules the nodes to minimize the average age at the destination by applying the notion of Markovian jump linear systems [12]. The approach undertaken in here is based on the discrete-time version of stochastic hybrid system (SHS) model for age analysis introduced in [13] and thus, is referred to as the *discrete-time* SHS (DT-SHS) model. We then derive the optimal link scheduling policy for the first network setting, whereas for the second setting, we optimize the AoI at the destination node numerically since the age-optimizing policies cannot be expressed in closed from.

The rest of this paper is organized as follows. In Section II, we introduce the DT-SHS model for age analysis in discrete-time Markovian systems. In Section III, the DT-SHS model is applied to two wireless relay network settings so as to analyze and optimize the AoI associated with the static link scheduling policies. The numerical results are discussed in Section IV. Finally, conclusions are drawn in Section V along with potential future directions.

II. Discrete-time Stochastic Hybrid Model

To be consistent with the SHS model in [13], consider a network with the discrete and continuous components of the system state at time slot k denoted as $q(k) \in \{1, 2, \ldots, Q\}$ and $\mathbf{x}(k) = [x_1(k)\ x_2(k)\ \cdots\ x_n(k)]^{\mathrm{T}} \in \mathbb{R}^n$, respectively, where Q is taken to be limited. In a discrete-time Markovian jump linear system, $q(k)$ evolves according to a discrete-time Markov chain (DTMC). Moreover, $\mathbf{x}(k)$ evolves linearly as:

$$\mathbf{x}(k+1) = \mathbf{A}_{l(k)} \cdot \mathbf{x}(k) + \mathbf{B}_{l(k)}, \tag{1}$$

where $l(k) \in \mathcal{L}$, defined as the Markovian jump parameter, is the transition at time k and belongs to the set of all transitions ($\mathcal{L}$) in the underlying DTMC[2]. In addition, $(\mathbf{A}_{l(k)}, \mathbf{B}_{l(k)})$ is the transition map corresponding to $l(k)$, indicating the linear mapping of the continuous component $\mathbf{x}(k)$ under transition $l(k)$. It is worth mentioning that similar to [13], there may be different transitions between two Markov chain states, each associated with a different transition map. Hereafter, we refer to the state of the DTMC simply as state unless otherwise stated. We define the state-indicator function as [12]:

$$I_i(k) = \begin{cases} 1, & q(k) = i, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

Consequently, the state-dependent mean vector $\mathbf{v}_i$ can be defined as:

$$\mathbf{v}_i(k) = \mathrm{E}\big[\mathbf{x}(k) I_i(k)\big]. \tag{3}$$

[2]In [12], $\mathbf{x}(k)$ evolves as $\mathbf{x}(k+1) = \mathbf{A}_{q(k)} \cdot \mathbf{x}(k)$, implying that all transitions initiating from state $q(k)$ have the same transition mapping $\mathbf{A}_{q(k)}$. Therefore, $q(k)$ represents the Markovian jump parameter. However, in (1), we consider a more general form of the linear transition mapping, where we include $\mathbf{B}_{l(k)}$ as well as a distinct transition mapping $\mathbf{A}_{l(k)}$ corresponding to each transition $l(k)$. Hence, in our model, $l(k)$ represents the Markovian jump parameter.

Corresponding to each transition l, there exists a directed edge (q_l, q'_l) and a transition probability p_l such that $q_l \xrightarrow{l} q'_l$ indicates the state change from q_l to q'_l under transition l. In order to derive $\mathbf{v}_i(k+1)$, we first re-write $\mathbf{x}(k+1)$ as:

$$\begin{aligned} \mathbf{x}(k+1) &= \sum_{l\in\mathcal{L}} \big(\mathbf{A}_l\mathbf{x}(k) + \mathbf{B}_l\big)\mathbb{1}\big\{l(k) = l\big\} && \text{(4a)} \\ &= \sum_{l\in\mathcal{L}} \big(\mathbf{A}_l\mathbf{x}(k) + \mathbf{B}_l\big)\mathbb{1}\big\{q(k) = q_l\big\}\mathbb{1}\{q_l \xrightarrow{l} q'_l\} && \text{(4b)} \\ &= \sum_j \sum_{l\in\mathcal{L}_j} \big(\mathbf{A}_l\mathbf{x}(k) + \mathbf{B}_l\big)\mathbb{1}\big\{q(k) = j\big\}\mathbb{1}\{j \xrightarrow{l} q'_l\}, && \text{(4c)} \end{aligned}$$

where $\mathcal{L}_j$ is the set of transitions initiating from state j and $\mathbb{1}\{\cdot\}$ is equal to one if its argument is true. Note that in (4b), $\mathbb{1}\{l(k) = l\}$ is replaced with $\mathbb{1}\{q(k) = q_l\}\mathbb{1}\{q_l \xrightarrow{l} q'_l\}$, meaning that transition l occurs if the current state is q_l and the state moves from q_l to q'_l under transition l. Using (4), $\mathbf{v}_i(k+1)$ can now be expressed as given below:

$$\begin{aligned} \mathbf{v}_i(k+1) &= \mathrm{E}\big[\mathbf{x}(k+1) I_i(k+1)\big] && \text{(5a)} \\ &= \sum_j \sum_{l\in\mathcal{L}_j} \mathrm{E}\big[\big(\mathbf{A}_l\mathbf{x}(k) + \mathbf{B}_l\big)\mathbb{1}\big\{q(k) = j\big\}\mathbb{1}\{j \xrightarrow{l} q'_l\} \\ &\qquad I_i(k+1)\big] && \text{(5b)} \\ &= \sum_j \sum_{l\in\mathcal{L}_{ji}} \mathrm{E}\big[\big(\mathbf{A}_l\mathbf{x}(k) + \mathbf{B}_l\big) I_j(k)\mathbb{1}\{j \xrightarrow{l} i\}\big] && \text{(5c)} \\ &= \sum_j \sum_{l\in\mathcal{L}_{ji}} \mathrm{E}\big[\big(\mathbf{A}_l\mathbf{x}(k) + \mathbf{B}_l\big) I_j(k)\big]\mathrm{E}\big[\mathbb{1}\{j \xrightarrow{l} i\}\big] && \text{(5d)} \\ &= \sum_j \sum_{l\in\mathcal{L}_{ji}} \mathbf{A}_l\mathbf{v}_j(k)p_l + \sum_j \sum_{l\in\mathcal{L}_{ji}} \mathbf{B}_l\mathrm{E}[I_j(k)]p_l, && \text{(5e)} \end{aligned}$$

where $\mathcal{L}_{ij}$ is the set of transitions from state i to state j. Note that in (5b), $\mathbb{1}\{j \xrightarrow{l} q'_l\}I_i(k+1)$ is zero for $q'_l \neq i$ and thus, is replaced with $\mathbb{1}\{j \xrightarrow{l} i\}$ in (5c). Also, (5d) is valid as $\mathbb{1}\{j \xrightarrow{l} i\}$ is independent of $x(k)$ and $I_j(k)$. Now, assuming that the system eventually becomes stable, the state-dependent mean vectors satisfy the following linear equations:

$$\begin{aligned} \mathbf{v}_i &= \sum_j \sum_{l\in\mathcal{L}_{ji}} \mathbf{A}_l\mathbf{v}_j p_l + \sum_j \sum_{l\in\mathcal{L}_{ji}} \mathbf{B}_l\mathrm{E}[I_j]p_l && \text{(6a)} \\ &= \sum_j \sum_{l\in\mathcal{L}_{ji}} \mathbf{A}_l\mathbf{v}_j p_l + \sum_j \sum_{l\in\mathcal{L}_{ji}} \mathbf{B}_l\pi_j p_l && \text{(6b)} \\ &= \sum_j \sum_{l\in\mathcal{L}_{ji}} \mathbf{A}_l\mathbf{v}_j p_l + \tilde{\mathbf{B}}_i, && \text{(6c)} \end{aligned}$$

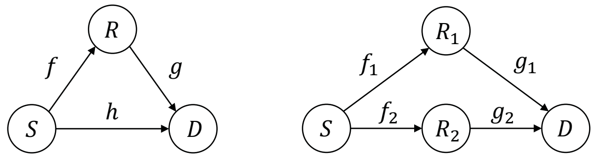
where π_i denotes the DTMC stationary probability of state i. Moreover, since $\tilde{\mathbf{B}}_i = \sum_j \sum_{l\in\mathcal{L}_{ji}} \mathbf{B}_l\pi_j p_l$, (6c) can be written in the following matrix form:

$$\tilde{\mathbf{v}} = (\mathbf{I}^{(nQ)} - \mathbf{R})^{-1}\tilde{\mathbf{B}}, \tag{7}$$

where $\mathbf{I}^{(y)}$ is the identity matrix of dimension $y \times y$, $\tilde{\mathbf{B}} = [\tilde{\mathbf{B}}_1^T\ \tilde{\mathbf{B}}_2^T\ \cdots\ \tilde{\mathbf{B}}_Q^T]^T$, $\tilde{\mathbf{v}} = [\mathbf{v}_1^T\ \mathbf{v}_2^T\ \cdots\ \mathbf{v}_Q^T]^T$, and $\mathbf{R}$ is a block matrix with $\mathbf{R}_{ij} \triangleq \sum_{l\in\mathcal{L}_{ji}} p_l\mathbf{A}_l$.

Note that the second moments of the age can be obtained by defining the state-indicator functions, $\mathbf{I}_i(k)$, and the state-dependent covariance vectors, $\mathbf{w}_i(k)$, respectively, as:

$$\mathbf{I}_i(k) = \begin{cases} \mathbf{I}^{(n)}, & q(k) = i, \\ 0, & \text{otherwise,} \end{cases} \tag{8}$$



(a) One relay and a direct link. (b) Two relays and no direct link.

Fig. 1. The two wireless relay networks of interest.

and $$\mathbf{w}_i(k)=\mathrm{E}\big[\mathbf{x}(k)\mathbf{x}^T(k)\mathbf{I}_i(k)\big]. \quad (9)$$

Similar to the earlier derivation of $\mathbf{v}_i$, $\mathbf{w}_i$ is obtained to be:

$$\mathbf{w}_i=\sum_j\sum_{l\in\mathcal{L}_{ji}}p_l\Big(\mathbf{A}_l\mathbf{w}_j\mathbf{A}_l^T+\mathbf{B}_l\mathbf{B}_l^T\pi_j+\mathbf{A}_l\mathbf{v}_j\mathbf{B}_l^T+\mathbf{B}_l\mathbf{v}_j^T\mathbf{A}_l^T\Big). \quad (10)$$

For sake of analysis, we define $x_m(k)$ as the age of node m at time k, where similar to [14], the age at node m is determined by the time elapsed since the generation of the most recent informative update, i.e. an age reducing update, received at node m. As a result, $x_m(k+1)$ is defined as:

$$x_m(k+1)=\begin{cases}a(k)+1, & \text{informative update is received,}\\ x_m(k)+1, & \text{otherwise,}\end{cases} \quad (11)$$

where $a(k)$ is the age of the update received at time k. Assuming that the update is received from node m', we have $a(k)=x_{m'}(k)$. Comparing (11) with (1) implies that $\mathbf{B}_l$ in (1) equals $\mathbf{1}_n$, where $\mathbf{1}_n$ is an $n\times 1$ all-one vector. Hence, $\tilde{\mathbf{B}}_i$ in (6) can be written as $\tilde{\mathbf{B}}_i=\mathbf{1}_n\sum_j\pi_j\sum_{l\in\mathcal{L}_{ji}}p_l=\mathbf{1}_n\sum_j\pi_j p_{ji}$, where p_{ji} is the transition probability from state j to state i in the DTMC. Given the global balanced equation of the DTMC, $\sum_j\pi_j p_{ji}=\pi_i$, we thus arrive at:

$$\tilde{\mathbf{B}}_i=\pi_i\mathbf{1}_n,\ \tilde{\mathbf{B}}=\big[\pi_1\mathbf{1}_n^T\ \pi_2\mathbf{1}_n^T\cdots\ \pi_Q\mathbf{1}_n^T\big]^T. \quad (12)$$

Finally, if we assume that the first entry of $\mathbf{x}(k)$, i.e. $x_1(k)$, is the age of the monitor, then the average AoI at the monitor can be formulated as follows:

$$\bar{x}_1=\sum_{i=1}^{Q}\mathbf{v}_i(1). \quad (13)$$

Based on the age definition in (11), the continuous version of (7) is $(\mathbf{D}-\mathbf{R}')^{-1}\tilde{\mathbf{B}}$, where $\tilde{\mathbf{B}}$ is computed in (12), $\mathbf{D}$ is an $nQ\times nQ$ block diagonal matrix with $\mathbf{D}_{ii}=(\sum_{l\in\mathcal{L}_i}\lambda_l)\mathbf{I}_n$, and $\mathbf{R}'$ is an $nQ\times nQ$ block matrix with $\mathbf{R}'_{ij}=\sum_{l\in\mathcal{L}_{ji}}\lambda_l\mathbf{A}_l$ and λ_l as the rate of transition l [14].

III. Age Analysis of Link Scheduling Policies in Wireless Relay Networks

In this section, we consider the two wireless relay network settings shown in Fig. 1. The first network is composed of a source (S), a relay (R), and a destination (D) node with a direct link between nodes S and D. The second network however, has two relay nodes, denoted by R_1 and R_2, and no direct link. In both networks, the channels between the nodes are modeled as packet erasure channels, i.e. the transmission in each channel is successful with a certain probability. Moreover, time is slotted and each slot duration is equal to one packet transmission time. In what follows, we use the DT-SHS model introduced in the preceding section to analyze and optimize the static link scheduling policies in the two relay network settings. The static link scheduling policies have no information on the state of the network. Also, considering the dynamics involved in the link scheduling policies or mobility of the nodes are beyond the scope of our analysis and are left for future works.

A. Setting I: One relay and direct link

As shown in Fig. 1a, the detection probabilities of channels $S\sim D$, $S\sim R$, and $R\sim D$ are given by h, f, and g, respectively. Nodes S and R are not allowed to transmit simultaneously in the same slot since it may result in collision at node D due to the presence of the direct link. In this regard, the static link scheduling policy schedules either node S or node R for transmission in each slot. More specifically, nodes S and R are scheduled for transmission with probabilities μ and $1-\mu$, respectively. Whenever permitted for transmission in a time slot, node S generates a new update and transmits it, where the update generation time is assumed to be negligible compared to its transmission time. The update of node S is received successfully at nodes D and R with probabilities h and f, respectively. Additionally, the transmissions of node S are not followed by ACK/NACK messages from nodes R and D since node S generates a new update at each of its scheduled time slots. On the other hand, node R is a half-duplex node which applies the lossy-LCFS policy to store the source updates, i.e. it only keeps the latest successfully received source update in its queue[3]. Upon being scheduled for transmission, node R transmits only if there is an update in its queue and remains silent, if otherwise. In the case when node R transmits its update successfully, node D then sends an ACK to node R at the end of the time slot. Subsequently, node R discards the update. Otherwise, it re-transmits the backlogged update.

In order to apply the DT-SHS model for AoI analysis, the discrete and continuous components of the system need to be modeled. The continuous component, $\mathbf{x}(t)$, is defined as $\mathbf{x}(t)=[x_D(t)\ x_R(t)]^T$, where $x_D(t)$ and $x_R(t)$ denote respectively, the ages at nodes D and R as defined in (11). To model the discrete component of the network, note that an update at node R can contribute to the age reduction at node D only if it is informative, i.e. since its reception at node R, no update has been received at node D via the direct link. Thus, the update status of node R should be included in the discrete component of the network state. Hence, let $q(t)\in\{\mathcal{N},\mathcal{I}\}$, where $\mathcal{I}$ indicates that there exists an informative update at node R, and $\mathcal{N}$ denotes that either node R is empty or the existing update is uninformative. The possible transitions of the network (l) and their corresponding probabilities (p_l), directed edges ((q_l,q_l')), and the transition map matrices ($\mathbf{A}_l$) are listed in Table I, where $\bar{y}\triangleq 1-y$. As an example, consider transition $l=0$, where node S is scheduled for transmission and its update is received successfully at nodes R and D. This implies that the received update at node R is uninformative. Thus, knowing that node R is in state $q_0=\mathcal{N}$, it remains in the same state. We also

[3] According to [15], the preemptive last-generated-first-served policy minimizes the age processes at all nodes in an information flow network when the packet transmission times over the links are exponentially distributed.

TABLE I
TRANSITIONS AND THEIR CORRESPONDING DIRECTED EDGES, PROBABILITIES, AND TRANSITION MAPS

l	(q_l, q'_l)	p_l	$\mathbf{A}_l$	l	(q_l, q'_l)	p_l	$\mathbf{A}_l$
0	$(\mathcal{N},\mathcal{N})$	$\mu h f$	$\begin{bmatrix}0&0\\0&0\end{bmatrix}$	6	$(\mathcal{I},\mathcal{N})$	$\mu h f$	$\begin{bmatrix}0&0\\0&0\end{bmatrix}$
1	$(\mathcal{N},\mathcal{N})$	$\mu h\bar{f}$	$\begin{bmatrix}0&0\\0&1\end{bmatrix}$	7	$(\mathcal{I},\mathcal{N})$	$\bar{\mu} g$	$\begin{bmatrix}0&0\\0&1\end{bmatrix}$
2	$(\mathcal{N},\mathcal{N})$	$\mu\bar{h}\bar{f}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	8	$(\mathcal{I},\mathcal{I})$	$\mu\bar{h}f$	$\begin{bmatrix}1&0\\0&0\end{bmatrix}$
3	$(\mathcal{N},\mathcal{N})$	$\bar{\mu}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	9	$(\mathcal{I},\mathcal{I})$	$\mu\bar{h}\bar{f}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$
4	$(\mathcal{N},\mathcal{I})$	$\mu\bar{h}f$	$\begin{bmatrix}1&0\\0&0\end{bmatrix}$	10	$(\mathcal{I},\mathcal{I})$	$\bar{\mu}\bar{g}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$
5	$(\mathcal{I},\mathcal{N})$	$\mu h\bar{f}$	$\begin{bmatrix}0&0\\0&1\end{bmatrix}$				

TABLE II
NEW TRANSITIONS, PROBABILITIES AND TRANSITION MAPS

l	p_l	A_l	l	p_l	A_l
0	$\mu h f$	$\begin{bmatrix}0&0\\0&0\end{bmatrix}$	3	$\mu\bar{h}\bar{f}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$
1	$\mu h\bar{f}$	$\begin{bmatrix}0&0\\0&0\end{bmatrix}$	4	$\bar{\mu} g$	$\begin{bmatrix}0&1\\0&1\end{bmatrix}$
2	$\mu\bar{h}f$	$\begin{bmatrix}1&0\\0&0\end{bmatrix}$	5	$\bar{\mu}\bar{g}$	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$

have $\mathbf{A}_0 = \left[\begin{smallmatrix}0&0\\0&0\end{smallmatrix}\right]$ since both nodes D and R have received an update with age one. The remaining cases are written in a similar manner. Using Table I, we have $p_{\mathcal{N}\mathcal{N}} = \mu h + \mu\bar{h}\bar{f} + \bar{\mu}$, $p_{\mathcal{N}\mathcal{I}} = \mu\bar{h}f$, $p_{\mathcal{I}\mathcal{N}} = \mu h + \bar{\mu}g$, and $p_{\mathcal{I}\mathcal{I}} = \mu\bar{h} + \bar{\mu}\bar{g}$. Thus, the steady-state probabilities of the DTMC are derived as $\pi_{\mathcal{N}} = p_{\mathcal{I}\mathcal{N}}/(p_{\mathcal{I}\mathcal{N}} + p_{\mathcal{N}\mathcal{I}})$ and $\pi_{\mathcal{I}} = 1 - \pi_{\mathcal{N}}$. From (7), (12), and (13), we can calculate the average AoI at node D, $\bar{x}_D$.

To solve (7), we need to obtain the inverse of $nQ \times nQ$, i.e. the 4×4 matrix in our problem. We thus, introduce a simpler DT-SHS model for our proposed network by reducing the number of states in DTMC to one. That is to say, we assume that when scheduled, node R always transmits an update, which is a fake one in the case of being empty. Moreover, the age of the fake update at time k is equal to $x_R(k)$. Then, in order to guarantee that the transmissions of node R, whether a fake one in the case of being empty or an uninformative one in the case of having an update, does not affect $x_D(k)$, we reset $x_R(k)$ to $x_D(k+1)$ whenever a new update is received at node D via the direct link, i.e. we reset $x_R(k)$ to one. In this case, x_R remains equal to x_D until a new informative update is received at node R. Consequently, in the new DT-SHS model, we have $q(t) = 0$ (i.e. there exists only one state) and $\mathbf{x}(t) = [x_D(t)\ x_R(t)]^T$. The new transitions and their corresponding probabilities, and transition maps are listed in Table II.

Proposition 1: In the first wireless relay network setting, the average AoI at node D is derived to be:

$$\bar{x}_D = \frac{\mu(\bar{g} - \bar{f}\,\bar{h}) + g}{\mu(1 - \bar{f}\,\bar{h})\big(\mu(h-g) + g\big)}. \tag{14}$$

Proof: We first compute $\mathbf{R} = \sum_{l=0}^{l=5} p_l \mathbf{A}_l$ using Table II. Then, using (7), (12), and (13), we obtain (14). □

Proposition 2: In the first wireless relay network setting, the age-optimal static link scheduling policy is derived to be:

$$\mu^* = \begin{cases} 1, & (\bar{g} = \bar{f}\,\bar{h},\ g \le 2h) \text{ or } \big(\bar{g} \ne \bar{f}\,\bar{h},\ g \le \frac{h(1-\bar{f}\,\bar{h})}{\bar{h}\,f}\big), \\ \frac{g}{2(g-h)}, & \bar{g} = \bar{f}\,\bar{h},\ g > 2h, \\ \frac{g}{\bar{g} - \bar{f}\,\bar{h}}\Big(\sqrt{\frac{\bar{h}f}{g-h}} - 1\Big), & \bar{g} \ne \bar{f}\,\bar{h},\ g > \frac{h(1-\bar{f}\,\bar{h})}{\bar{h}f}. \end{cases} \tag{15}$$

Proof: See Appendix A. □

B. Setting II: Two relays with no direct link

We now consider the network in Fig. 1b, where nodes S and D communicate only through the relays. The relays are assumed to be full duplex, i.e. they can transmit and receive simultaneously. The detection probabilities of channels $S \sim R_i$ and $R_i \sim D$ ($i \in \{1,2\}$) are f_i and g_i, respectively. At each time slot, node S transmits a new update with probability one. The update is received at the end of slot at nodes R_1 and R_2 with probabilities f_1 and f_2, respectively, and can be transmitted only in the next slots. On the other hand, the relays apply the lossy-LCFS policy when buffering the updates in their queues. In each slot, the static policy schedules one of the relays for transmission; it chooses node R_1 with probability μ and node R_2 with probability $1-\mu$. Each relay receives an ACK from node D in the case of successful transmission. If an ACK is not received, the relays keep their current updates for probable re-transmissions.

Next, we derive the average AoI at node D by applying the DT-SHS model. Here, the continuous component of the network is $\mathbf{x}(t) = [x_D(t)\ x_1(t)\ x_2(t)]$, where $x_i(t)$ is the age of node R_i, as per the definition in Section II. Recall that each relay may have a different status including being empty, backlogged with an informative, or an uninformative update. While it is impossible for both relays to possess uninformative updates, it is however possible that both relays have informative updates. To avoid including all these states in the DTMC, we adopt an approach similar to that in Section III-A. We assume that whenever the relay with a lower age, i.e. the relay which has or had (in the case of a prior successful transmission) the most informative update, transmits successfully (a fake transmission when being empty), the age of the other relay resets to the age of node D (i.e., $x_D(k+1)$). In this case, the transmissions of the relay node with the uninformative (real or fake) update does not affect x_D. Thus, we include the relay with lower age in the state and model the discrete component as $q(t) = \{1,2\}$, where state $i \in \{1,2\}$ indicates that $x_i(k) \le x_j(k)$, $j \ne i$.

In the introduced DTMC, the state is $i \in \{1,2\}$ as long as a new update has not entered node R_j ($j \ne i \in \{1,2\}$). Hence, the transition probabilities can be expressed as $p_{11} = \bar{f}_2$, $p_{12} = f_2$, $p_{22} = \bar{f}_1$, and $p_{21} = f_1$. Consequently, $\pi_1 = \frac{f_1}{f_1+f_2}$ and $\pi_2 = \frac{f_2}{f_1+f_2}$. With μ_i defined as the transmission probability of node R_i in each slot, we have:

$$\mu_i = \begin{cases} \mu, & i = 1, \\ \bar{\mu}, & i = 2. \end{cases} \tag{16}$$

The set of transition probabilities corresponding to transitions l with $q_l = q'_l = i$, where $i \in \{1,2\}$, can now be written as:

$$\{p_l | (q_l, q'_1) = (i,i)\} = \{p_{ii}xy | x \in \{\mu_1 g_1, \mu_1 \bar{g}_1, \mu_2 g_2, \mu_2 \bar{g}_2\}, y \in \{f_i, \bar{f}_i\}\}. \tag{17}$$

In fact, x in (17) indicates which relay transmits and whether its transmission is successful or not. Similarly, y specifies whether a new update is generated at node R_i or not. If we let $[\mathbf{A}_l]_m$ denote the m-th row of $\mathbf{A}_l$, then $[\mathbf{A}_l]_1$ corresponding to each p_l in (17) can be written as (note that $[\mathbf{A}_l]_1$ indicates the mapping of x_D):

$$[\mathbf{A}_l]_1 = \begin{cases} \mathbf{e}_1, & x \in \{\mu \bar{g}_1, \bar{\mu} \bar{g}_2\}, \\ \mathbf{e}_2, & x = \mu g_1, \\ \mathbf{e}_3, & x = \bar{\mu} g_2, \end{cases} \tag{18}$$

where $\mathbf{e}_n$ is a 1×3 vector with the n-th entry equal to one and all other entries equal to zero. The first case in (18) indicates that x_D increases by one when no update is received successfully at node D. In the second and third cases, x_D resets to the age of nodes R_1 and R_2, respectively, due to a successful transmission. Here, $[\mathbf{A}_l]_{i+1}$ is given as:

$$[\mathbf{A}_l]_{i+1} = \begin{cases} \mathbf{0}, & y = f_i, \\ \mathbf{e}_{i+1}, & y = \bar{f}_i, \end{cases} \tag{19}$$

where $\mathbf{0}$ is a 1×3 zero vector. Equation (19) informs that if a new update is generated at node R_i, then x_i resets to zero and otherwise, increases by one. Finally, $[\mathbf{A}_l]_{j+1}$ $(j \neq i, j \in \{1,2\})$ is calculated as below:

$$[\mathbf{A}_l]_{j+1} = \begin{cases} \mathbf{e}_{i+1}, & x = \mu_i g_i, \\ \mathbf{e}_{j+1}, & x \neq \mu_i g_i. \end{cases} \tag{20}$$

The first case in (20) states that if node R_i with a lower age transmits successfully, the age of node R_j resets to $x_i(k)+1$, i.e. $x_D(k+1)$; else, it increases by one. The set of transition probabilities from state i to state j when $i \neq j$ are obtained similar to (17), except that p_{ii} is substituted with p_{ij}. Also, $[\mathbf{A}_l]_1$ and $[\mathbf{A}_l]_{i+1}$ are derived exactly as (18) and (19), respectively. However, $[\mathbf{A}_l]_{j+1} = \mathbf{0}$ since a new update is generated at node R_j as a result of transition to state j. Note that in this section, we optimize $\bar{x}_D$ numerically as the closed-form expressions for μ^* cannot be derived.

IV. Numerical Results and Discussions

This section presents some numerical results to illustrate the impact of varying channel detection probabilities on the average AoI at the destination node. Simulation results are reported for some figures to validate our analytical formulation. Conducted in MATLAB environment, the simulated results have been obtained after sufficiently long runs to ensure that the network reaches the steady-state.

In Fig. 2, the average AoI at node D in the first relay network setting is plotted against the scheduling parameter, μ, for $f = 1$ and $h = 0.1$. As evident in the figure, when the detection probability of $R \sim D$ channel (i.e. g) is low, it is optimal to receive all updates from the source S (i.e. $\mu^* = 1$). As g increases however, it is optimal to receive some of the updates from node R (i.e. $\mu^* < 1$). Moreover, the AoI improvement due to cooperation also increases with g.

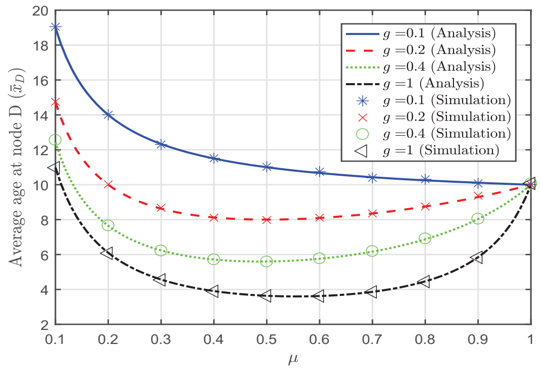


Fig. 2. Average AoI at node D in terms of the scheduling parameter, μ, for $h = 0.1$ and $f = 1$.

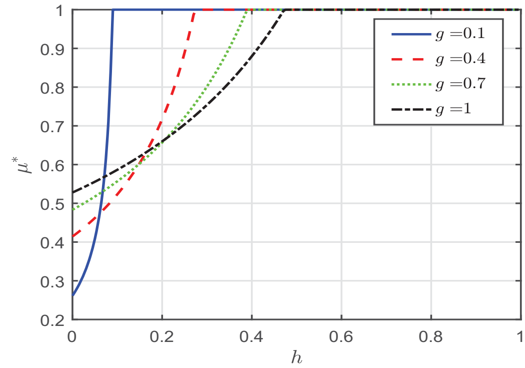


Fig. 3. Optimal μ value in terms of the detection probability of the direct link, h, for $f = 0.8$.

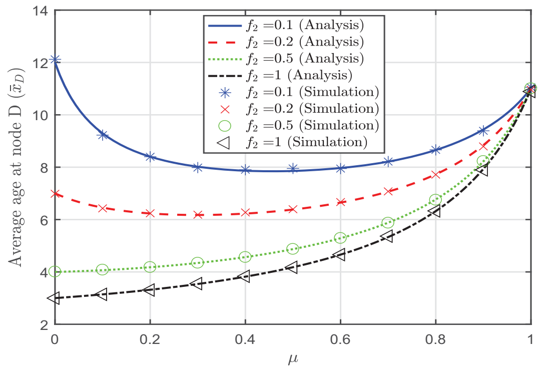


Fig. 4. Average AoI in terms of the scheduling parameter, μ, for $f_1 = 0.1$, $g_1 = 1$, and $g_2 = 0.5$.

Fig. 3 shows the optimal value of μ in the first network setting in terms of the detection probability of the direct link, h, at different g values. As seen in the figure, μ^* increases with h since by increasing h, the informative updates can be received from the direct link with a higher probability. Note that at $h = 0$, μ^* increases with g. In fact, as g grows, the static policy decides to activate node S for transmission more frequently, i.e. letting more updates enter node R, since the updates can be transmitted successfully at node R with a higher probability.

The average AoI at node D in the second network setting versus the scheduling parameter, μ, is depicted in Fig. 4 at different f_2 values. Here, f_1, g_1, and g_2 are set to 0.1, 1, and 0.5, respectively. We observe that when f_2 is high, the AoI at node D increases with μ since in this case, node R_2, which

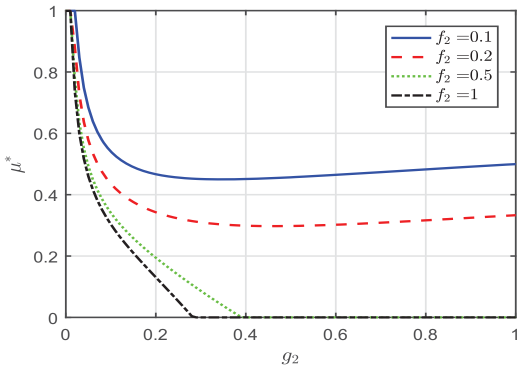


Fig. 5. Optimum average AoI at node D in terms of g_2 for $f_1 = 0.1$ and $g_1 = 1$.

has fresher updates ($f_2 > f_1$), obtains less opportunities for transmission. However, when f_2 becomes lower and closer to f_1, the AoI decreases with μ at first since node R_1, having a better downlink channel, transmits with a higher probability. It then, increases with μ, leading to $\mu^* < 1$. This reveals that even when $f_1 \approx f_2$ and $g_1 > g_2$, it is optimal to exploit both the relays and not only node R_1.

In Fig. 5, the optimal scheduling parameter, μ^*, in the second network setting is shown against the detection probability of $R_2 \sim D$ channel, g_2, where the channels related to node R_1 are assumed to be fixed ($f_1 = 0.1, g_1 = 1$). One can observe that when $g_2 = 0$, we have $\mu^* = 1$. However, when g_2 increases from zero, μ^* decreases very fast, giving more opportunities for transmission to node R_2. This reveals that although node R_2 has a weak link to destination, its cooperation can serve effective in improving AoI.

V. Conclusion

In this paper, we have proposed the DT-SHS model for analyzing AoI in discrete-time Markovian systems. Then, we have applied the DT-SHS model for analyzing and optimizing the average age associated to static link scheduling policies in two wireless relay network settings. To simplify the derivation of DTMCs in the DT-SHS model, we have assumed that whenever the relay with a lower age transmits successfully, the age of other nodes resets to the age of the destination node. Numerical results have been presented to investigate the effect of relaying on age improvement. We have shown that even relays with very low quality links can be effective in AoI improvement at the destination. Some interesting extensions to this work include investigating the relationship between the amount of AoI improvement and the number of relays, efficiency of dynamic scheduling policies, and impact of user mobility in wireless relay networks.

Appendix A
Proof of Proposition 2

The derivative of $\bar{x}_D$ is given as:

$$\frac{d\bar{x}_D}{d\mu} = \frac{(\bar{g} - \bar{f}\,\bar{h})(h-g)\mu^2 + 2g(h-g)\mu + g^2}{-(1-\bar{f}\,\bar{h})\mu^2\big(\mu(h-g)+g\big)^2}. \quad (21)$$

In what follows, we consider the cases $\bar{g} \neq \bar{f}\,\bar{h}$ and $\bar{g} = \bar{f}\,\bar{h}$.

- When $\bar{g} \neq \bar{f}\,\bar{h}$, the derivative in (21) has roots $\mu_1 = \frac{g}{\bar{g}-\bar{f}\,\bar{h}}\big(-1+\frac{\sqrt{f\bar{h}(g-h)}}{h-g}\big)$ and $\mu_2 = \frac{g}{\bar{g}-\bar{f}\,\bar{h}}\big(-1-\frac{\sqrt{f\bar{h}(g-h)}}{h-g}\big)$. If $h > g$, no real roots exist, $\bar{x}_D$ is decreasing in $\mu \in (0,1]$, and thus, $\mu^* = 1$. If $h < g$ and $\mu_1, \mu_2 \in \mathbb{R}$, then the roots are re-written as $\mu_1 = \frac{g}{\bar{g}-\bar{f}\,\bar{h}}\big(-1-\sqrt{\frac{f\bar{h}}{g-h}}\big)$ and $\mu_2 = \frac{g}{\bar{g}-\bar{f}\,\bar{h}}\big(-1+\sqrt{\frac{f\bar{h}}{g-h}}\big)$. Thus, μ_1 is either greater than 1 if $1-\bar{f}\,\bar{h} < r$ or less than -1, if otherwise. However, $\mu_2 > 0$. In order to have $\mu_2 < 1$ and thus, $\mu^* = \mu_2$, it can be shown by some mathematical manipulation that g should satisfy $g > \frac{h(1-\bar{f}\,\bar{h})}{\bar{h}\,f}$. If this inequality does not hold, then $\bar{x}_D$ is decreasing in $[0,1]$ thus, resulting in $\mu^* = 1$. Note that $\frac{h(1-\bar{f}\,\bar{h})}{\bar{h}\,f} > h$ and $g > \frac{h(1-\bar{f}\,\bar{h})}{\bar{h}\,f}$ yields $h < g$, i.e. the necessary condition for having real roots.
- When $\bar{g} = \bar{f}\,\bar{h}$, the unique positive root is $\frac{g}{2(g-h)}$. To have $\mu^* = \mu_1$, the inequality $\mu_1 < 1$ should hold (i.e. $g > 2h$). Otherwise, $\mu^* = 1$.

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