Decentralized Equilibrium Seeking of Joint Routing and Destination Planning of Electric Vehicles: A Constrained Aggregative Game Approach

Babak Ghaffarzadeh Bakhshayesh, *Graduate Student Member, IEEE*,
and Hamed Kebriaei, *Senior Member, IEEE*

***Abstract*— Increasing the penetration of electric vehicles (EVs) in public transportation, which is also sped up by governments' carbon net-zero policies, will significantly increase the demand for electricity. Therefore, when we face with a large population of selfish EV users, we need a coordination mechanism to manage both the traffic congestion and electricity resource limitations. This paper introduces a novel aggregative game model where heterogeneous EVs simultaneously plan their parking lot as their destination (usually accompanied by battery charging) and the route to the destination. The cost function of users consists of factors such as traveling time, variable costs of congestion and electricity demand, and tolling which is imposed to satisfy coupling constraints such as roads' capacity and stations' power capacity. Since the users are selfish and do not reveal their objectives and personal constraints, we propose a privacy preserving decentralized algorithm with a traffic coordinator and multiple stations' coordinators for generalized Nash equilibrium (GNE) seeking of the game model. Only aggregate information such as traffic on the road and stations' energy demand are available to the traffic coordinator and charging stations' coordinators, respectively. We show that the proposed aggregative game admits a unique variational generalized Nash equilibrium (v-GNE). Then, using the theory of variational inequality (VI), we show that the proposed decentralized algorithm converges to the unique v-GNE of the game. Finally, we carry out comprehensive simulation studies on a simulated Savannah city model to compare and evaluate the proposed method.**

***Index Terms*— Aggregative games, electric vehicles (EVs), charging stations, routing and destination planning, generalized Nash equilibrium (GNE), decentralized algorithm.**

NOMENCLATURE

EV	Electric Vehicle.
VI	Variational Inequality.
v-GNE	Variational Generalized Nash Equilibrium.
$\mathcal{N}$	set of users ($i \in \mathcal{N} = \{1, \ldots, N\}$).
$\mathcal{V}$	set of transportation nodes ($v \in \mathcal{V} = \{1, \ldots, V\}$).
$\mathcal{E}$	set of road segments ($(u, v) \in \mathcal{E} = \{1, \ldots, E\}$).
$\mathcal{D}$	set of destinations ($d \in \mathcal{D} = \{1, \ldots, D\}$).
r_i^e	probability of choosing road $e \in \mathcal{E}$ by user i.
t_i^d	probability of choosing destination $d \in \mathcal{D}$ by user i.
r_i/t_i	routing/station strategy of user i.
x_i/x	$x_i = col(r_i, t_i)$ $/x = [x_i]_{i=1}^N$.
ω_i	monetary value of time for user i.
σ_e	expected flow of vehicles on road $e \in \mathcal{E}$.
a_e	road $e \in \mathcal{E}$ congestion-free travel time.
b_e	road $e \in \mathcal{E}$ capacity.
θ, ξ	regulating parameter of travel time function.
s_e	non-EV vehicles flow on road $e \in \mathcal{E}$.
α_i/β_i	monetary value of user i routing/station preference.
φ_d	expected energy demand at station $d \in \mathcal{D}$.
q_i	inelastic energy demand of user i.
ρ_d/p_d	fixed/variable price of station $d \in \mathcal{D}$.
κ_d	capacity of energy production of station $d \in \mathcal{D}$.
$\mathcal{X}_i$	strategy set of user i.
S	modified incidence matrix of transportation network.
o_i	origin of user i ($o_i \in \mathcal{V}$).
$\mathcal{C}$	coupling constraints set ($\mathcal{C} = \mathcal{C}^r \times \mathcal{C}^t$).
c_d^t	upper bound of station $d \in \mathcal{D}$ capacity constraint.
c_e^r	upper bound of road $e \in \mathcal{E}$ capacity constraint.
l_e	travel time function of road $e \in \mathcal{E}$.
λ_t^d	surcharge at station $d \in \mathcal{D}$.
λ_r/λ_t	toll vector of roads/stations.

Manuscript received March 18, 2021; revised August 10, 2021; accepted October 20, 2021. This work was supported in part by the Institute for Research in Fundamental Sciences (IPM) under Grant CS 1400-4-451. The Associate Editor for this article was C. Lv. *(Corresponding author: Hamed Kebriaei.)*

Babak Ghaffarzadeh Bakhshayesh is with the School of Electrical and Computer Engineering, College of Engineering, University of Tehran, Tehran 14399-57131, Iran (e-mail: b.ghaffarzadeh@ut.ac.ir).

Hamed Kebriaei is with the School of Electrical and Computer Engineering, College of Engineering, University of Tehran, Tehran 14399-57131, Iran, and also with the School of Computer Science, Institute for Research in Fundamental Sciences (IPM), Tehran 19538-33511, Iran (e-mail: kebriaei@ut.ac.ir).

Digital Object Identifier 10.1109/TITS.2021.3123207

I. INTRODUCTION

A. Motivation

WITH the emergence of new technologies such as Google Maps, Waze, Plugshare, and Crowdless apps, users have access to real-time information in order to choose their route and destination [1]. Studies have shown that users participate as an active role in the infrastructure so that, if the state of transportation network changes, users quickly respond to that change and alter their decisions [2].

One of the decision-making problems is raised when the users choose their route and destination based on their preferences and data provided by online platforms, such as congestion on roads and crowdedness of destination. The competitive nature of such a decision-making problem becomes apparent when the users are willing to reach the lowest crowded destination (e.g., charging station) in the shortest time. This situation arises in problems like population migration [3], pandemic mitigation [4], supermarket selection [5], Autonomous mobility-on-demand [6], public parking [7], and Electric Vehicle (EV) charging station selection that crowdedness could affect electricity prices and waiting times [8].

Clearly, routing and destination planning are not two disjoint decision variables since the selection of a destination (e.g., charging station) can limit the routing choices and vice versa. On the other hand, increasing the number of EVs and limited number of public charging facilities with limited electric power resources emerges some challenges for both the transportation network and charging stations' operation. In such a situation, when the users are willing to drive to the nearest station, the electricity demand can significantly increase in some stations [9]. Therefore, it is essential to study and control such an effect due to the limited resources and facilities.

B. Literature

The research lines on transportation network and EV charging problem include charging services network planning and operations [10], designing the network price mechanisms [11], [12], and user equilibrium analysis on how individual EVs choose their routes and charging station.

In [13], an equilibrium model is proposed to study the dependency of battery energy consumption and recharging time on traffic times; however, regardless of the demand in the charging stations, the price of electricity at charging stations is considered to be fixed. [14] investigates the equilibrium of the transportation network, assuming that electric vehicles can be charged through wireless chargers while driving and thus, EVs do not need to choose a charging station. More recently, [15] studied the charging station network equilibrium using a threshold-based structure to incorporate the time value in EVs' preferences. Also, in [16], authors used a behavioral framework based on prospect theory to study charging strategies of EV users and derived an upper bound for the price of anarchy. In [12], authors consider a non-separable congestion game to incorporate the charging cost of EVs. The game determines a specific charging cost for each type of user depending on the users' routing strategies. It is also assumed that users delegate control over their charging operations to the central operator. In [17], authors investigated a graphical game-theoretical model to schedule the electric vehicles whose primary concern is the latency. Assuming the latency time depends on the neighbor vehicles of each EV, the correlated equilibrium point of the game is computed. In the proposed scheme, EV users send their preferences to a trustworthy platform and execute the recommended actions by the platform. The authors of [18] analyzed the fixed point of the transportation and power network by solving the traffic management and optimal power flow problems, sequentially. In this way, by solving the transportation problem, energy demand at each station is computed, and then, solving power system optimization, the electricity price at each station is determined. This process continues until a fixed point is attained. In [19], a comprehensive review is provided on transportation network models for charging station selection strategies. The approaches given in [12]–[19] need users to enumerate all feasible paths, which leads to solving a mixed-integer linear program. Therefore, due to the NP-hardness of the problem, their algorithm may not be computationally efficient for implementation in large-scale networks. Authors in [20] adopted a cake-cutting game model to demonstrate competition among Public vehicle groups to supply public transportation demand. Each public vehicle group plans to schedule a portion of public vehicles and their charging strategies in the transportation network. However, they did not study the routing strategy of vehicles in the transportation network. In [21], the authors considered a network flow model to obtain a socially optimal solution of joint routing and charging station planning of EVs in a transportation network. For this purpose, a multi-nominal logit-based model was proposed to determine demand in stations. However, the model parameters need to be calibrated empirically, and heterogeneity of users, which is closer to reality, was not investigated.

In the aforementioned studies, a central operator determines the equilibrium point of the system by having full information of users, and hence, by increasing the number of EVs, conventional game-theoretical models are not computationally tractable. Nevertheless, no decentralized computational algorithm is proposed in order to enable heterogeneous selfish users to reach equilibrium point of the system using common real-time information (like traffic on the roads and prices at stations) without using the direct recommendation of a central coordinator.

This paper aims to analyze the equilibrium point of joint routing and destination selection problem of EVs users, considering the traveling time, users' preferences, stations' crowdedness, and electricity price. To address this problem, we focus on a certain type of non-cooperative games, which is known as aggregative games [22]. In aggregative games, the coupling effect of players' decisions is modeled as an aggregate term. Aggregative games are well suited for developing efficient decentralized algorithms for computation of the Nash equilibrium point of the game [23]. Also, they preserve users' privacy because selfish users do not need to reveal their cost functions and personal constraints info to the aggregators and their strategies to the other players. Aggregative games have received much attention in recent years, and they have been applied to various domains like smart grid [24], network congestion [25], routing games [23], residential charging planning for EVs [26], [27], etc. Recently, several studies analyzed the aggregative games in the presence of coupling constraints among users' decision space [23], [28], [29]. We consider the effect of coupling constraints among the users raised by the limited capacity of infrastructures [30], [31] (e.g., the capacity of roads and stations' power) which is omitted in most of related studies. We formulate the network equilibrium problem with heterogeneous players as generalized Nash equilibrium Problem [32].

generalized Nash equilibrium (GNE) can be characterized as a variational inequality problem which is called variational generalized Nash equilibrium (v-GNE) [33].

C. Contributions

We extend the theory of constrained aggregative game to examine two interdependent aggregate terms: congestion on the roads and total demand at the charging stations. The coupling of these two aggregate terms comes from the simultaneous routing and destination planning of EVs in our approach. Then, the existence and uniqueness of the v-GNE of the constrained aggregative game are proven.

After that, motivated by the prevalence of smartphones among people and awareness of users from crowdedness in the charging stations and roads, we propose a decentralized algorithm to compute the v-GNE. In this algorithm, in addition to the variable energy price, each station sets a price associated with a Lagrange multiplier to incentivize users' behaviors; furthermore, the users do not reveal their objectives and preferences. Convergence of the algorithm to the v-GNE is also proven.

The contributions of the paper can be summarized as follows:

- We propose a novel game-theoretic model for joint planning of route and charging station of EVs based on the (modified) edge formulation of routing game that reduces the problem complexity.
- We extend the theory of constrained aggregative game by considering two interdependent aggregate terms to simultaneously include the congestion on roads and energy demand in stations.
- We incorporate the coupling constraints among users to encompass power energy and road capacity limitation, and then a decentralized algorithm is proposed to seek the game's v-GNE.
- We prove the existence and uniqueness of the proposed constrained game's v-GNE and convergence of the decentralized algorithm to the v-GNE.

D. Organization of Paper

The remainder of the paper is organized as follows: Section II provides the decision-making model and highlights the relation between routing and destination selection. In Section III, we derive v-GNE point of the game and establish a unique v-GNE. In section IV, we propose a decentralized Nash seeking algorithm to solve the problem as an aggregative game with coupling constraints. We also show the convergence of the algorithm to the v-GNE. Section V presents an illustrative application of the proposed scheme. Section VI concludes the paper.

Notation: $\mathbb{R}$ and $\mathbb{R}_{\geq 0}$ denotes the set of real and non-negative real numbers. Given N vectors $x_1, \ldots, x_N$, the column augmented vectors of x_i is defined as: $\mathrm{col}(x_1, \cdots, x_N) = [x_i]_{i=1}^{N} = [x_1^\top, \ldots, x_N^\top]^\top$. $\|x\|$ is the euclidean norm of vector x. For closed set $\mathcal{X} \subseteq \mathbb{R}^n$, mapping $\mathrm{proj}_{\mathcal{X}}(x) : \mathbb{R}^n \to \mathcal{X}$ denotes projection onto set $\mathcal{X}$ (i.e. $\mathrm{proj}_{\mathcal{X}}(x) = \arg\min_{y \in \mathcal{X}} ||y - x||$). The symbol $\perp$ is used to denote perpendicular vectors. $\mathbb{1}_N = [1, \ldots, 1]^\top$ and $\mathbb{0}_N = [0, \ldots, 0]^\top$, where $\mathbb{1}_N, \mathbb{0}_N \in \mathbb{R}^N$. I_N is an identity matrix of size N. $\mathbf{0}_{m \times n}$ presents a $m \times n$ matrix with all elements equal to zero. $A \otimes B$ represents the Kronecker product. $\nabla_x f(x) \in \mathbb{R}^{d \times n}$ denotes gradient of function $f(x) : \mathbb{R}^n \to \mathbb{R}^d$. For matrix A, $[A]_{i,j}$ is an element on row i and column j. $\mathrm{diag}(A_1, \ldots, A_N) = \mathrm{diag}\{A_i\}_{i=1}^{N}$ denote a block-diagonal matrix with matrix A_i on the ith block. We indicate matrix $A \in \mathbb{R}^{n \times n}$ as a positive definite (semi-definite) matrix with $A \succ 0$ ($A \succcurlyeq 0$). Mapping $F : \mathcal{X} \in \mathbb{R}^n \to \mathbb{R}^n$ is: 1. monotone (strongly monotone with constant $c > 0$) on $\mathcal{X}$ if, for every $x, y \in \mathcal{X}, x \neq y$, $(F(x) - F(y))^\top (\text{x-y}) \geq 0 \left(\geq c\|\text{x-y}\|^2\right)$, and 2. Lipschitz continues on $\mathcal{X}$ with constant L if, for any $x, y \in \mathcal{X}, x \neq y$, $L \geq 0$, $\|F(x) - F(y)\| \leq L\,\|\text{x-y}\|$.

II. System Model

Consider a transportation network with N EV driving users and D charging stations whose sets are denoted by $\mathcal{N}$ and $\mathcal{D}$, respectively. Charging stations correspond to parking lots of vehicles equipped with charging stations that provide the same service for the EV users. Each user aims to reach one of these destinations to receive the charging service.

We consider a closed area of a transportation network modeled as a strongly connected directed graph $(\mathcal{V}, \mathcal{E})$. The node set $\mathcal{V} = \{1, \ldots, V\}$ represents the physical intersections in the transportation network, destinations $\mathcal{D} \subseteq \mathcal{V}$, and EVs' origins. The edge set $\mathcal{E} = \{1, \ldots, E\} \subseteq \mathcal{V} \times \mathcal{V}$ indicates the road segments.

A. Individual User Cost Function

Each user $i \in \mathcal{N}$ decides his/her destination and routing, simultaneously. Here we adopt the probabilistic strategy for each user i denoted by $x_i = \mathrm{col}(r_i, t_i) \in \mathbb{R}_{\geq 0}^{E+D}$, where $t_i = \left[t_i^d\right]_{d=1}^{D}$ represents the destination selection probability distribution of user i over $\mathcal{D}$, and $r_i = \left[r_i^e\right]_{e=1}^{E}$ stands for the routing probability of user i, which is a probability distribution over $\mathcal{E}$. Also, $r = [r_i]_{i=1}^{N}$ and $t = [t_i]_{i=1}^{N}$ are destination and route selection vectors of users, respectively. The users selfishly seek a destination with the lowest crowding and price with the best quality of service while minimizing their traveling time.

Naturally, each user does not have access to decision information of other individual users. Therefore, we assume that the real-time congestion information services provide only the aggregate information of route flows and the energy demand of charging stations to all users.

The expected cost function of each user $i \in \mathcal{N}$ can be defined as follows:

$$J_i(r_i, t_i, \sigma(r), \varphi(t)) = U_i\,(r_i, t_i) + \omega_i\; C_i^{travel} + C_i^{service}, \quad (1)$$

where U_i indicates the cost of deviation from user's preferred choices, C_i^{travel} is the expected travel time experienced by EV, $C_i^{service}$ is the expected service cost (e.g., electricity, parking) paid by user, and $\omega_i \in \left[\underline{\omega},\ \bar{\omega}\right]$ is a weight factor that represents the monetary value of time for EV. In what follows, we discuss these terms in detail.

1) Travel Time: Expected travel time user i experienced by going from its origin to the chosen charging station is given by:

$$C_i^{travel}\left(r_i,\sigma(r)\right)=\sum_{e=1}^{E} l_e\left(\sigma_e\left(r^e\right)\right)r_i^e \tag{2}$$

$l_e(\cdot):\mathbb{R}_{\geq 0}\rightarrow\mathbb{R}_{>0}$ is a strictly-increasing function of the total flow of vehicles on road $e\in\mathcal{E}$. We assume a general latency function offered by The Bureau of Public Roads to capture traffic congestion as a function of total vehicles in the corresponding road, and it is as follows:

$$l_e(\sigma_e(r^e))=a_e\left(1+\theta\left(\frac{\sigma_e(r^e)}{b_e}\right)^{\xi}\right), \tag{3}$$

where a_e can be thought of as congestion-free travel time in road e, which is proportional to the length of roads and free-flow speed. Capacity of road is modeled by b_e, which is proportional to the width of the road. Also, $\xi\geq 1$ and $\theta>0$ are parameters of the proposed BPR latency function [4], [34]. We assume that only EVs are smart and equipped to send and receive the required signals to compute the best solution. On the other side, we assume that the aggregate behavior of non-electric vehicles (Non-EVs) is predictable using available data of road congestion [35]. The total vehicle flow on road e is caused by EVs and non-EVs, denoted by $\sum_{i=1}^{N} r_i^e$ and s_e, respectively. In other words, $\sigma_e(r^e)$ is the total expected flow of vehicles on road $e\in\mathcal{E}$ that is defined as:

$$\sigma_e(r^e)=s_e+\sum_{i=1}^{N} r_i^e. \tag{4}$$

We also define $\sigma(r)=\left[\sigma_e\left(r^e\right)\right]_{e=1}^{E}$ and $r^e=\left[r_i^e\right]_{i=1}^{N}$ as the flow vector of roads and users' strategy of choosing road $e\in\mathcal{E}$.

2) User Preference: The cost associated with deviating from user preferences is formalized as:

$$U_i=\frac{\alpha_i}{2}\|r_i-\tilde{r}_i\|^2+\frac{\beta_i}{2}\left\|t_i-\tilde{t}_i\right\|^2, \tag{5}$$

where $\tilde{t}_i=\left[\tilde{t}_i^d\right]_{d=1}^{D}\in[0,\ 1]^D$, $\sum_{d=1}^{D}\tilde{t}_i^d=1$ and $\tilde{r}^i=\left[\tilde{r}_i^e\right]_{e=1}^{E}\in[0,\ 1]^E$, denote the preferred destination and path for user i based on his/her previous travel experiences, respectively. Also, $\alpha_i\in[\underline{\alpha},\ \bar{\alpha}]$ and $\beta_i\in[\underline{\beta},\ \bar{\beta}]$ are the constant weight parameters which showing the monetary value of their preferences.

3) Service Price: The expected service cost afforded to user i is modeled as follows:

$$C_i^{service}\left(t_i,\varphi_d(t)\right)=\sum_{d=1}^{D}\left[q_i\,p_d(\varphi_d(t^d))+\rho_d\right]t_i^d, \tag{6}$$

where $q_i\in[\underline{q},\ \bar{q}]$ is user i's energy inelastic demand, ρ_d is the once paid fixed cost of parking at the station d, and $p_d(\cdot):\mathbb{R}_{\geq 0}\rightarrow\mathbb{R}_{>0}$ is the per-unit cost of charging at the station d which depends on total energy demand at that station, defined as:

$$p_d(\varphi_d(t^d))=\delta_d\left(\frac{\varphi_d\left(t^d\right)}{\kappa_d}\right) \tag{7}$$

where $\delta_d>0$ is predefined pricing constant, κ_d is the capacity of electricity production [26], and $\varphi_d(t^d)$ is the expected total demand at station $d\in\mathcal{D}$, defined as follows:

$$\varphi_d(t^d)=\sum_{i=1}^{N}\tilde{q}_i^d, \tag{8}$$

where $\tilde{q}_i^d=q_i t_i^d$ is the expected energy consumption of user i at station $d\in\mathcal{D}$, and $t^d=\left[t_i^d\right]_{i=1}^{N}$. Note that, if destination d is not equipped with EV charger or an agent is not willing to charge, the parameters δ_d and q_i will be equal to zero, respectively. In both cases, the agent will only pay for the fixed cost ρ_d. We also define $\varphi(t)=\left[\varphi_d\left(t^d\right)\right]_{d=1}^{D}$. The station's energy cost for EVs is affected by the demand at the station that depends on the users' route choice strategies. The explicit relation of these two is given in what follows.

B. Constraints

The transportation network is modeled as a multi-commodity network flow problem characterized by its node-edge incidence matrix $\Delta\in\{0,\ 1,\ -1\}^{V\times E}$, where:

$$[\Delta]_{v,e}=\begin{cases}1 & \text{if edge } e \text{ enters node } v\\ -1 & \text{if edge } e \text{ leaves node } v\\ 0 & \text{otherwise.}\end{cases} \tag{9}$$

We assume that the last D rows in matrix Δ correspond to destination nodes, which are decision variables. To ensure the feasibility of problem as the user $i\in\mathcal{N}$ who starts from origin $o_i\in\mathcal{V}\setminus\mathcal{D}$, arrives at a feasible service location $d\in\mathcal{D}$ with probability t_i^d, each user must satisfy the following constraint:

$$\sum_{e:(u,v)\in\mathcal{E}} r_i^e-\sum_{e:(v,w)\in\mathcal{E}} r_i^e=\begin{cases}-1 & \text{if}\quad v=o_i\\ t_i^d & \text{if}\quad v=d\\ 0 & \text{otherwise,}\end{cases}\qquad \forall v\in\mathcal{V},\quad \forall d\in\mathcal{D}, \tag{10}$$

where $e:(u,v)\in\mathcal{E}$ indicates the edge e that starts from node u and ends at node v. It can be interpreted as the probability of exiting the origin node is equal to 1, and the probability of entering the dth destination is equal to t_i^d.

Also, due to the technical issues like EV's charger connector type compatibility and range anxiety, we consider that each user i can only go to a specific set of charging stations denoted by $\mathcal{D}_i\subseteq\mathcal{D}$. Therefore, the strategy of user $i\in\mathcal{N}$, have to satisfy the following individual constraint:

$$\mathcal{X}_i:=\left\{x_i\in[0,1]^{E+D}|Sx_i=m_i,\,t_i^d=0\ \forall d\in\mathcal{D}\setminus\mathcal{D}_i\right\} \tag{11}$$

where $m_i=[m_i^v]_{v=1}^{V}\in\{0,\ -1\}^V$ is such that $m_i^v=-1$ if node v is the origin of user i and $m_i^v=0$, otherwise. Modified incidence matrix $S \in \{0,\ 1,\ -1\}^{V\times(E+D)}$ has the following structure:

$$S = \left[\begin{array}{c|c} & \mathbf{0}_{(\text{V-D})\times D} \\ \Delta_{V\times E} & \text{-----} \\ & -I_D \end{array}\right] \tag{12}$$

Constraint (11) shows an explicit relation between choosing routes and charging stations for each user. Under this constraint, destinations are considered as decision variables of users, in contrast to the predetermined destinations in the edge formulation of routing games.

C. Infrastructure Constraints

We also consider the physical constraints on shared roads and charging stations, which impose coupling constraints on the users' strategies. Hence, we define the coupling constraints $\mathcal{C}^r$ and $\mathcal{C}^t$, as follows:

$$\mathcal{C}^t = \left\{t \in \mathbb{R}^{MD} \,\middle|\, \varphi_d\left(t^d\right) \le c^t_d, \forall d \in \mathcal{D}\right\} \tag{13a}$$

$$\mathcal{C}^r = \left\{r \in \mathbb{R}^{ME} \mid \sigma_e\left(r^e\right) \le c^r_e, \forall e \in \mathcal{E}\right\} \tag{13b}$$

where $c^t = \left[c^t_d\right]_{d=1}^{D}$ and $c^r = \left[c^r_e\right]_{e=1}^{E}$ stand for the maximum capacity of the stations and roads, respectively [30] and $s = [s_e]_{e=1}^{E}$. The constraint $\mathcal{C}^t$ implies that each station $d \in \mathcal{D}$ can't respond to load more than c^t_d, and the constraint $\mathcal{C}^r$ implies that each road $e \in \mathcal{E}$ has a maximum capacity of c^r_e in which, the number of vehicles in road e cannot exceed from c^r_e. Each charging station would be responsible for satisfying its constraint. One can represent these two coupling constraints together as $\mathcal{C} = \mathcal{C}^r \times \mathcal{C}^t$.

III. Game Theoretical Formulation

The cost function of each user in (1) is affected by the aggregative terms (8) and (4) through the price of energy and road flows, respectively. Additionally, according to (13), there is a coupling among the strategy set of users. Such interaction and coupling among users impose a non-cooperative aggregative game among EVs. The aggregative game among the users can be formalized by introducing the set of players, cost functions, individual constraints, and coupling constraints, respectively as follows:

$$\mathcal{G} := \left(\mathcal{N},\ \{J_i\}_{i\in\mathcal{N}}, \{\mathcal{X}_i\}_{i\in\mathcal{N}}, \mathcal{C}\right). \tag{14}$$

The feasible collective set $\mathcal{X}$ defined as:

$$\mathcal{X} := \prod_{i=1}^{N} \mathcal{X}_i \bigcap \mathcal{C}. \tag{15}$$

Therefore, the collective strategy profile of the users is $x := [x_i]_{i=1}^{N} \in \mathbb{R}_{\ge 0}^{N(E+D)}$. We also let $\Theta(x) = \mathrm{col}(\sigma(r), \varphi(t))$ be the vector of collective aggregative terms.

Definition 1 (Generalized Nash Equilibrium): A set of strategies $\bar{x} := [\bar{x}_i]_{i=1}^{N}$ is a generalized Nash equilibrium (GNE) of game $\mathcal{G}$ if:

$$J_i\left(\bar{x}_i, \Theta\left(\bar{x}\right)\right) \le J_i\left(x_i, \Theta\left(x_i, \bar{x}_{-i}\right)\right), \quad \forall x\ s.t.\ (x_i, \bar{x}_{-i}) \in \mathcal{X},\ \forall i \in \mathcal{N} \tag{16}$$

where, $\bar{x}_{-i} = (\bar{x}_1, \ldots, \bar{x}_{i-1}, \bar{x}_{i+1}, \ldots, \bar{x}_N), \forall i \in \mathcal{N}$. □

This means that no agent has an incentive for unilateral deviation from $\bar{x}_i$ to another feasible strategy at an equilibrium state given the aggregated strategies of the other users.

Definition 2 (Variational Inequality): Given a set $\mathcal{X}$ and mapping $F : \mathcal{X} \to \mathbb{R}^n$, a vector $\bar{x} \in \mathcal{X}$ solves the variational inequality problem, denoted by VI$(\mathcal{X}, F)$ if $F(\bar{x})^\top(y - \bar{x}) \ge 0, \forall y \in \mathcal{X}$. □

For the game $\mathcal{G}$ in (14), the pseudogradient mapping $F : \mathcal{X} \to \mathbb{R}^n$ is defined as:

$$F\left(x\right) := \left[\nabla_{x_i} J_i\left(x_i, \Theta\left(x\right)\right)\right]_{i=1}^{N} \tag{17}$$

which is constructed by accumulating the subgradients of the agents' cost functions with respect to their strategies. As we interest that users incur an equal penalty to satisfy coupling constraints (i.e. coupling constraints have the same Lagrange multiplier for all the users), we focus on a subset of GNEs, which is known as a variational generalized Nash equilibrium (v-GNE). Such a problem can be reformulated as a solutions of variational inequality VI$(\mathcal{X}, F)$ [33].

Lemma 1: The mapping $F : \mathcal{X} \to \mathbb{R}^n$ of the game $\mathcal{G}$ in (14) is strongly monotone if the following condition holds:

$$\min_{e\in\mathcal{E}}\left(s_e - \frac{(\xi-1)\,N\,\bar{\omega}}{8\,\underline{\omega}}\right) \ge 0. \tag{18}$$

□

Proof: Based on equation (17), the mapping F of game $\mathcal{G}$ in (14) can be written as the following three summands:

$$F = \underbrace{\begin{bmatrix} \alpha_i(r_i - \tilde{r}_i) \\ \beta_i(t_i - \tilde{t}_i) \end{bmatrix}_{i=1}^{N}}_{\text{Term 1}} + \underbrace{\begin{bmatrix} \omega_i\ l(\sigma(r)) \\ q_i\ p\left(\varphi(t)\right) + \rho \end{bmatrix}_{i=1}^{N}}_{\text{Term 2}} + \underbrace{\begin{bmatrix} \omega_i\ \nabla_\sigma l(\sigma(r))\ r_i \\ (q_i)^2\ \nabla_\varphi p(\varphi(t))\ t_i \end{bmatrix}_{i=1}^{N}}_{\text{Term 3}} \tag{19}$$

that $l\left(\sigma(r)\right) = \left[l_e\left(\sigma_e(r^e)\right)\right]_{e=1}^{E}$, $p\left(\varphi(t)\right) = \left[p_d\left(\varphi_d(t^d)\right)\right]_{d=1}^{D}$ and $\rho = [\rho_d]_{d=1}^{D}$. Since the set $\prod_{i=1}^{N} \mathcal{X}_i$ is compact and coupling constraints functions in (13) are affine, it follows from [36, Proposition 2.3.2] that a continuously differentiable mapping G is monotone if and only if $\nabla_x G(x) \succeq 0$ for all $x \in \mathcal{X}$. Term 1 in F is linear with positive coefficient and so it is strongly monotone. For each d and e, functions $p_d(\cdot)$ and $l_e(\cdot)$ are increasing and hence, Term 2 is monotone. To analyze the Term 3, and for ease of notation, we define $q = [q_i]_{i=1}^{N}$, $\Omega = \mathrm{diag}\{\omega_i\}_{i=1}^{N}$, $Q_2 = \mathrm{diag}\{(q_i)^2\}_{i=1}^{N}$. We note that for all e and d, we have: $\nabla_z l_e(z)\big|_{z=\sigma_e(r^e)} = l'_e(\sigma_e)$, $\nabla_y p_d(y)\big|_{y=\varphi_d(t^d)} = p'_d(\varphi_d)$, $\nabla^2_z l_e(z)\big|_{z=\sigma_e(r^e)} = l''_e(\sigma_e)$ and $\nabla^2_y p_d(y)\big|_{y=\varphi_d(t^d)} = p''_d(\varphi_d)$.

To prove monotonicity of the Term 3, it suffices to show that:

$$\nabla_{\begin{bmatrix} r \\ t \end{bmatrix}} \left\{ \begin{bmatrix} \left[\ \omega_i\ \nabla_\sigma l(\sigma(r))\ r_i\right]_{i=1}^{N} \\ \left[(q_i)^2\ \nabla_\varphi p(\varphi(t))\ t_i\right]_{i=1}^{N} \end{bmatrix} \right\}$$

$$= \mathrm{diag}\ \left(\mathrm{diag}\left\{\omega_i \mathrm{diag}\left\{l'_e\left(\sigma_e\right)\right\}_{e=1}^{E}\right\}_{i=1}^{N},\ \mathrm{diag}\left\{(q_i)^2 \mathrm{diag}\left\{p'_d\left(\varphi_d\right)\right\}_{d=1}^{D}\right\}_{i=1}^{N}\right)$$

$$+\mathrm{diag}(\mathbb{1}_N \otimes \left(\left[\mathrm{diag}\left\{\omega_i l_e''(\sigma_e)\, r_i^e\right\}_{e=1}^{E}\right]_{i=1}^{N}\right)^\top,$$

$$q \otimes \left(\left[\mathrm{diag}\{(q_i)^2 p_d''(\varphi_d)\; t_i^d\}_{d=1}^{D}\right]_{i=1}^{N}\right)^\top) \succeq 0 \quad (20)$$

Since the permutation on a matrix doesn't change its positive (or negative) semi definiteness, we apply a permutation matrix to (20), so that the resulting matrix is decomposed to some block-diagonal matrices. The permutation matrix P is defined as:

$$P = \begin{bmatrix} \left[[\vec{\mathbf{1}}_{e+(i-1)E}^\top]_{i=1}^{N}\right]_{e=1}^{E} \\ \left[[\vec{\mathbf{1}}_{d+N\times E+(i-1)D}^\top]_{i=1}^{N}\right]_{d=1}^{D} \end{bmatrix}, \quad (21)$$

where in matrix P, $\vec{\mathbf{1}}_i$ is the ith canonical vector basis.

We define $\forall e \in \mathcal{E}$, $\iota_e(r^e) = \left[\omega_i r_i^e\right]_{i=1}^{N}$ and $\forall d \in \mathcal{D}$, $u_d(t^d) = [(q_i)^2 t_i^d]_{i=1}^{N}$. After employing the permutation matrix P, the matrix (20) will be permuted to:

$$P\nabla_{\left[\begin{smallmatrix} r\\ t\end{smallmatrix}\right]}\left\{\begin{bmatrix} [\ \omega_i\ \nabla_\sigma l(\sigma(r))\ r_i]_{i=1}^{N} \\ \left[(q_i)^2\ \nabla_\varphi p(\varphi(t))\ t_i\right]_{i=1}^{N}\end{bmatrix}\right\} P^\top$$
$$= \mathrm{diag}(\mathrm{diag}\left[l_e'(\sigma_e)\,\Omega\right]_{e=1}^{E} + \mathrm{diag}\left[l_e''(\sigma_e)\,\iota_e \mathbb{1}_N^\top\right]_{e=1}^{E},$$
$$\mathrm{diag}\left[p_d'(\varphi_d)\, Q_2\right]_{d=1}^{D} + \mathrm{diag}\left[p_d''(\varphi_d)\, u_d q^\top\right]_{d=1}^{D}) \quad (22)$$

According to the block-diagonal structure of the matrix in (22), to prove its positive semi definiteness, it suffices to show following conditions:

$$l_e'(\sigma_e)\,\Omega + l_e''(\sigma_e)\,\iota_e(r^e)\mathbb{1}_N^\top \succeq 0 \quad \forall e \in \mathcal{E} \quad (23a)$$
$$p_d'(\varphi_d)\, Q_2 + p_d''(\varphi_d)\, u_d(t^d)q^\top \succeq 0 \quad \forall d \in \mathcal{D} \quad (23b)$$

As the electricity price function is linear, the second derivative of price function disappears, so for the condition (23b), it is sufficient that function to be a increasing function (i.e., for all d, $p_d'(\varphi_d) \geq 0$). For the latency functions (3); since the strategy variables r are bounded, one can compute $\lambda_{min}\left(\iota_e(r^e)\,\mathbb{1}_N^\top + \mathbb{1}_N \iota_e^\top(r^e)\right)/2$, which is the minimum eigenvalue of matrix $\iota_e(r^e)\,\mathbb{1}_N^\top$. As a result, for each e we have $\min\limits_{r^e\in[0,1]^N} \lambda_{\min}\left(\iota_e(r^e)\,\mathbb{1}_N^\top + \mathbb{1}_N\iota_e^\top(r^e)\right)/2 \geq \frac{-\bar{\omega}N}{8}$. So for (23a), we have the following conditions:

$$\min_{\substack{e\in\mathcal{E}\\ \chi\in[0,N]}} (l_e'(\chi)\,\underline{\omega} - \frac{\bar{\omega}\, l_e''(\chi)\, N}{8}) \geq 0 \quad (24)$$

Replacing travel time function $l_e(r)$ from (3) and simplifying (24), we have:

$$\min_{e\in\mathcal{E}} (s_e - \frac{(\xi-1)\, N\, \bar{\omega}}{8\, \underline{\omega}}) \geq 0. \quad (25)$$

By noting that the summation of some strongly monotone and monotone mappings is strongly monotone, as long as the condition (18) holds, lemma 1 is proved. ■

Remark 1: If the latency function is considered to be linear as in [37], we have $\xi = 1$, and hence, (25) is always satisfied. □

Proposition 1: Any solution to VI $(\mathcal{X}, F)$ is a Nash equilibrium of the game $\mathcal{G}$ in (4). □

Proof: For each $i \in \mathcal{N}$ the set $\mathcal{X}_i$ in (11) is compact and convex, and coupling constraints functions in (13) are affine; the set $\mathcal{X}$ in (15) is nonempty and satisfies slater's constraint qualification. Also, for each i, the cost function $J_i(x_i,\ \Theta(x))$ in (1) is convex in x_i and twice continuously differentiable in x for any feasible strategy set $\mathcal{X}$. Convexity is directly derived by showing $\nabla_x F(x) \succeq 0$ in Lemma 1. Under this situation, we employ [38, Proposition 12.4] to establish that any solution of VI $(\mathcal{X}, F)$ is a Nash equilibrium of game $\mathcal{G}$. ■

The next theorem demonstrates the existence and uniqueness of variational equilibrium.

Theorem 1: If condition (18) holds, the game $\mathcal{G}$ in (14) admits a unique variational Nash equilibrium. □

Proof: According to [38, Proposition 12.11], variational inequality VI $(\mathcal{X}, F)$ has a unique solution if Mapping F is strongly monotone on set $\mathcal{X}$. As long as the condition (18) in Lemma 1 holds, the theorem is proved. ■

IV. Decentralized Game-Theoretical Framework

To solve the generalized equilibrium of game $\mathcal{G}$ in a decentralized fashion, we introduce new variables $\lambda = \mathrm{col}(\lambda_r, \lambda_t)$, where λ_r and $\lambda_t = [\lambda_t^d]_{d=1}^{D}$ are dual variables of infrastructure constraints (13b) and (13a), respectively, and also we append their non-negativity condition $\lambda_r \in \mathbb{R}_{\geq 0}^{E}$ and $\lambda_t \in \mathbb{R}_{\geq 0}^{D}$. We define extended variational inequality VI $(\mathcal{Y},\ T)$ as follows:

$$T(x,\ \lambda_r,\ \lambda_t) := \begin{bmatrix} F(x) + \left[\nabla_{x_i}\left(q_i(t_i^\top \lambda_t) + r_i^\top \lambda_r\right)\right]_{i=1}^{N} \\ \left[c_e^r - \sigma_e(r^e)\right]_{e=1}^{E} \\ \left[c_d^t - \varphi_d(t^d)\right]_{d=1}^{D} \end{bmatrix} \quad (26a)$$

$$\mathcal{Y} := \prod_{i=1}^{N} \mathcal{X}_i \times \mathbb{R}_{\geq 0}^{E} \times \mathbb{R}_{\geq 0}^{D} \quad (26b)$$

As can be seen in the new variational inequality problem (26), the coupling constraints are eliminated. λ_t and λ_r can also be interpreted as tolls (surcharge) paid by users for charging their vehicles and traveling on roads, respectively. In the following proposition, we show the equivalence of VI $(\mathcal{Y},\ T)$ and VI $(\mathcal{X},\ F)$.

Proposition 2: $\mathrm{col}(\bar{x}, \bar{\lambda}_r, \bar{\lambda}_t)$ is a solution of VI $(\mathcal{Y},\ T)$ if and only if $\bar{x}$ is a solution of VI $(\mathcal{X},\ F)$. □

Proof: Since sets $\mathcal{X}$ and $\mathcal{Y}$ satisfy Slater's constraint qualification, by using [39, Theorem 3.1], we can establish there exists $\overline{\lambda} = \mathrm{col}(\overline{\lambda_r}, \overline{\lambda_t})$ such that $\mathrm{col}(\bar{x}, \bar{\lambda}_r, \bar{\lambda}_t)$ solves VI $(\mathcal{X},\ F)$ if and only if $\overline{x}$ is the solution of VI $(\mathcal{X},\ F)$. ■

As we showed the equivalency of problems VI $(\mathcal{X},\ F)$ and VI $(\mathcal{Y},\ T)$, we use VI $(\mathcal{Y},\ T)$ to compute the equilibrium point of $\mathcal{G}$ in (14). Algorithm 1 solves VI $(\mathcal{Y},\ T)$ based on asymmetric projected gradient method [36]. In this algorithm, each user locally updates its decision variables based on aggregate information on road flow (σ), destination congestion (φ), and the dual variables (surcharges) associated with the coupling constraints (λ_r and $\lambda_t = [\lambda_t^d]_{d=1}^{D}$) to minimize its cost function. At each iteration, the transportation system operator gathers and broadcasts the aggregative traffic condition

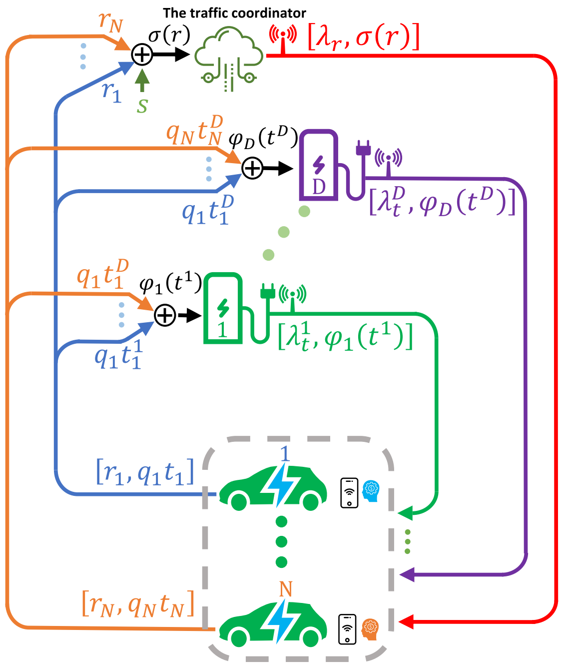


Fig. 1. The schematic of information flows of the system.

Algorithm 1 Aggregative Game

Initialization: $k \leftarrow 1$, $\tau > 0$, $t_i^{(1)} \in \mathbb{R}^D_{\geq 0}$, $r_i^{(1)} \in \mathbb{R}^E_{\geq 0}$, $\lambda_t^{(1)} \in \mathbb{R}^D_{\geq 0}$, $\lambda_r^{(1)} \in \mathbb{R}^E_{\geq 0}$.

Iteration k:

Aggregation:

$$\sigma^{(k)} \leftarrow s + \sum_{i \in \mathcal{N}} r_i^{(k)}$$

$$\varphi_d^{(k)} \leftarrow q_i t_i^{d\,(k)}, \forall d \in \mathcal{D}$$

Planning: for each $i \in \mathcal{N}$:

$$x_i^{(k+1)} \leftarrow \mathrm{proj}_{\mathcal{X}_i}[x_i^{(k)} - \tau(\nabla_{x_i} J_i(x_i^{(k)}, \Theta(x^{(k)})) + \nabla_{x_i}(q_i (t_i^{(k)})^\top \lambda_t^{(k)} + (r_i^{(k)})^\top \lambda_r^{(k)}))]$$

Dual variables (Surcharge) update: for each $d \in \mathcal{D}$:

$$R_d{}^{(k+1)} \leftarrow 2\varphi_d(t^{d\,(k+1)}) - \varphi_d(t^{d\,(k)})$$

$$\lambda_t^{d\,(k+1)} \leftarrow \mathrm{proj}_{\mathbb{R}_{\geq 0}}[\lambda_t^{d\,(k)} - \tau(c_d^t - R_d{}^{(k+1)})]$$

$$\lambda_r^{(k+1)} \leftarrow \mathrm{proj}_{\mathbb{R}^E_{\geq 0}}[\lambda_r^{(k)} - \tau(c^r - 2\sigma(r^{(k+1)}) + \sigma(r^{(k)}))]$$

$k \leftarrow k + 1$

---

of roads (σ) to users. It also updates and informs the dual variable vector (road's toll) λ_r based on violation of defined road capacity constraints c^r in (13b). Also, each charging station d independently collects and announces the expected energy demand of users ($\varphi_d(t^d)$) at that station. Then, each station d updates and broadcasts the dual variable (surcharge) λ_t^d based on violation of defined energy capacity constraint c_d^t in (13a). Users and aggregators keep updating these variables until the convergence of the algorithm occurs. Fig. 1 illustrates the schematic of information flows among the transportation coordinator and charging stations with EVs.

τ is the fixed step size of gradient learning. The convergence of Algorithm 1 to v-GNE of $\mathcal{G}$ is studied in the following Proposition.

Proposition 3: If condition (18) holds and the step size τ is chosen sufficiently small, then Algorithm 1 converges to a unique v-GNE of game $\mathcal{G}$ in (14). □

Proof: Based on Theorem 1, $\mathcal{G}$ admits a unique variational Nash equilibrium. Also, based on Lemma 1, F is strongly monotone on set $\mathcal{X}$, and it is Lipschitz in the same bounded set since the cost functions in (1) are continuously differentiable. Besides, the set of coupling constraints in (13) are affine. Therefore, according to [23, Theorem 3], there exists a $\bar{\tau} > 0$ such that for any $0 < \tau < \bar{\tau}$, Algorithm 1 converges to the v-GNE of the game $\mathcal{G}$ in (14). ■

Remark 2: Since algorithm 1 globally converges to the game's unique Nash equilibrium, if users miss some information during communication with aggregators, users' strategies will converge to the unique Nash equilibrium as soon as they retrieve the latest data from aggregators of the game (14). □

V. Simulation Results

In this part, we apply techniques developed in Section IV to model the competition among users who want to charge their EVs. We consider the transportation network and Tesla Supercharger charging stations of Historic District of Savannah, US, Georgia State, to illustrate user equilibrium. We model the main roads as edges and the intersection of the roads as nodes. The road network includes 58 bi-directional edges and 31 nodes, and six nodes of charging stations distributed in the network, as depicted in Fig. 5. We consider one-hour time intervals to solve the problem, and We use the linear form of latency function in (3) with $\xi = 1$ and $\theta = 4$ [37]. Free-flow travel time on roads ($\frac{\text{length}}{\text{free-flow speed}}$) derived based on vehicles speed in the uncongested condition, which is 30 km/h on a typical day in Savannah city. We also assume the capacity of roads heuristically computed based on $b_e = 20\frac{\text{vehicle}}{\text{km}} \times$ (free-flow speed) by assuming 20 vehicles per kilometer traveling on the uncongested condition. Exogenous congestion caused by non-EV users at different hours on roads has been approximately selected from typical traffic conditions data on Sundays in Savannah city, available in Google Maps (i.e., variables s_e has been set between 100 $\frac{\text{vehicles}}{\text{hr}}$ and 300 $\frac{\text{vehicles}}{\text{hr}}$). The maximum capacity of each road is $c_e = 3b_e$. We consider users' origins are distributed uniformly across the city. Users' various preferences parameters and energy demand are taken from a uniform distribution $\omega_i \sim U(18, 70)$ (in \$/hr), $q_i \sim U(20, 70)$ (in kWh), and we set parameters α_i and β_i values to 0.8\$ for all users. The parameters concerned with charging stations are listed in Table I. The station's energy price is chosen to roughly match the average energy price in the state of Georgia, which is 9.8 ¢/kWh in June 2020 [40].

EVs availability on each hour is taken from the danish driving patterns on different hours [11]. The availability of the EVs in various hours (time slots) is depicted in Fig. 2.

We solve the problem for each time slot, and for instance, the expected demand of station 6 for the twenty-four-hour period is depicted in Fig. 3. Fig. 3 also compares different

TABLE I
CHARGING STATIONS' COEFFICIENTS

Station	1	2	3	4	5	6
δ_d (\$/kWh)	0.07	0.09	0.055	0.09	0.1	0.07
κ_d (MWh)	0.725	0.725	0.725	0.725	0.725	0.725
c_d^t (MWh)	1.4	1.5	0.8	1	1.3	0.9
ρ_d (\$)	0	0	0	1	0	0

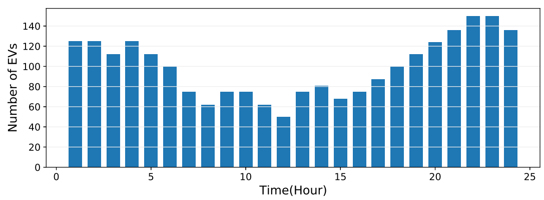


Fig. 2. EVs arrival.

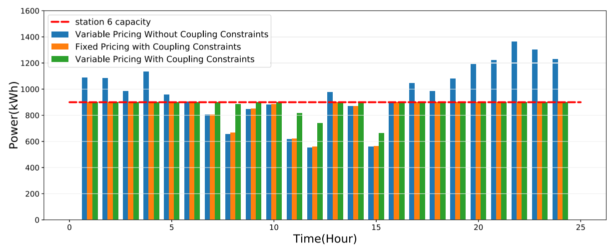


Fig. 3. The expected energy demand of station 6 in a day.

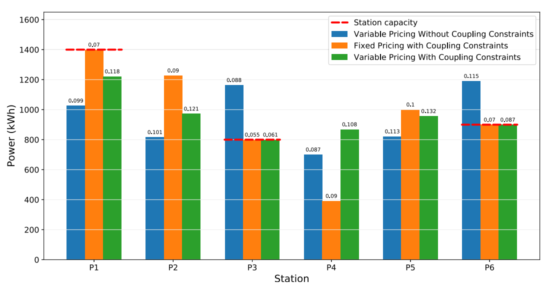


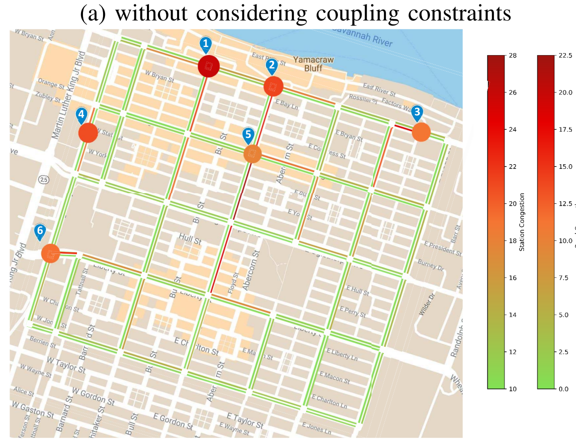
Fig. 4. Expected energy demand and energy price of each station considering variable pricing with and without coupling constraint, and fixed pricing with coupling constraint.

methodologies, including variable pricing with and without coupling constraints, and fixed pricing with coupling constraints. We observe in times that the energy demand in the city is high, by ignoring the effect of coupling constraints, the expected demand of station 6 violates its power capacity limit.

We specifically analyze the problem at 8 pm, where 125 EVs with different preferences parameters want to charge their vehicles. Fig. 4 shows stations' expected demand with their variable prices ($\frac{\$}{\text{kWh}}$) comparing different methodologies.

As seen in Fig. 4, by neglecting the coupling constraints in the decision-making procedure, the expected energy demands of stations 3 and 6 violate the energy capacity of related stations, which shows the inefficiency of neglecting the coupling constraints. We have also examined the effectiveness of variable pricing over fixed pricing policy. We observe that, by employing variable pricing, expected energy demands of

(a) without considering coupling constraints



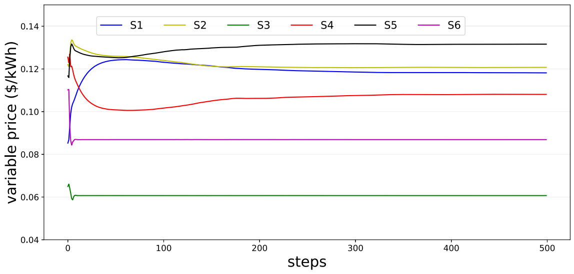
(b) with considering coupling constraints

Fig. 5. Transportation network and Charging stations equilibrium.

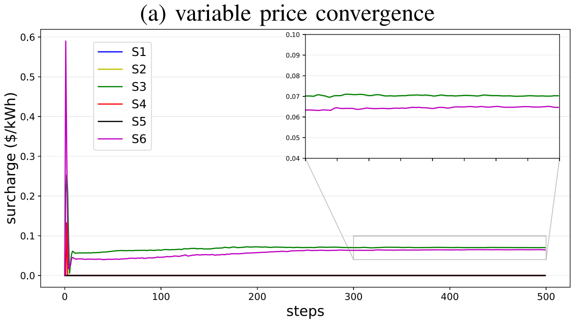
EVs are better distributed among different stations than fixed pricing.

Fig. 5 depicts the network equilibrium with and without considering coupling constraints. We see in Fig. 5 that considering coupling constraints also affects the mobility pattern of users while we notice congestion nearby stations 3 and 6 is reduced. Fig. 6 shows the convergence of each station's variable price and each stations' toll (i.e., surcharge). To see the effect of tolling, if we look again at Fig. 4 since stations 3 and 6 reach their load capacity, they penalize users, motivating users to alter their decisions for satisfying their power capacity constraints. Generally, EVs are willing to go to the nearest station with the lowest energy price paid. In the uncoupled equilibrium condition shown in Fig. 5a, due to network structure, stations 3, 4, and 6 are closer to more users. Hence, we expect higher demand in these charging units. Further, as the price of energy in stations 3 and 6 are lower than station 4, so there is more interest in stations 3 and 6, and there is more congestion on their nearby roads. However, in the more realistic case shown in Fig. 5b, the stations' power capacity is considered and hence, as the popular stations' capacity is completed, users choose other stations whose power capacities are not full. It is also can be observed from Fig. 5 that in areas where charging stations are densely located, more traffic occurs.

Also, due to the heterogeneity of users, it is obvious that they show various behavior. For example, we compare two



(a) variable price convergence



(b) surcharge convergence

Fig. 6. Variable price and surcharge (dual variable) convergence considering coupling constraints.

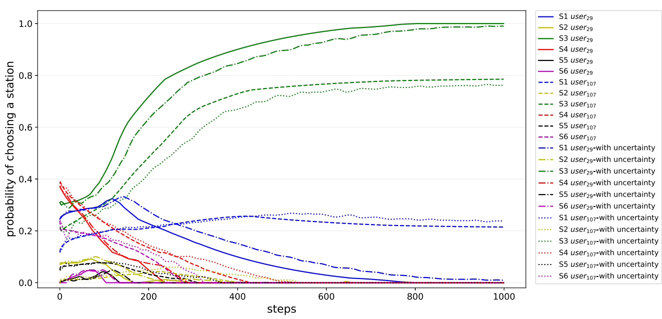


Fig. 7. User 29 and user 107 charging station planning convergence with and without uncertainty in aggregation.

EVs, user 29 and user 107, from the same origin nearby station 3. User 29 has parameters $\omega_{29} = 63.7$ (in \$/hr) and $q_{29} = 20.97$ (in kWh), and user 107 has parameters $\omega_{107} = 26.66$ and $q_{107} = 30.76$. So user 29 cares more than user 107 for travel time. Fig. 7 shows the destination selection decision convergence of these two users. As seen in Fig. 7, user 29, who cares about travel time, chooses the nearest station with a probability of 1. In contrast, user 107, who cares less about spent time, chooses the nearest station with a probability of 0.78. In order to examine the scenario where stations are unable to gather precise values of expected demand at each step, we assume bounded randomness in stations' aggregative terms. Therefore, for each station we have: $\hat{\varphi}_d^{(k)} = \varphi_d{}^{(k)} + \epsilon_t^{d\,(k)}$, where $\epsilon_t^{d\,(k)}$ is a random variable with uniform distribution. We assume that the uncertainty at each station and each step is a random variable with $\epsilon_t^{d\,(k)} \sim U\,(0, 80)$. As it can be seen from Fig. 7, the users' strategies converge close to the scenario with perfect information.

Finally, we examine the effect of probabilistic strategies of users on violating the coupling constraint when the users realize their random strategies to select only one station.

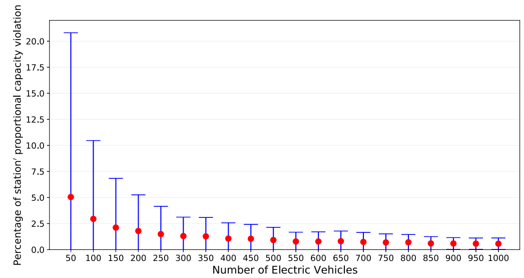


Fig. 8. Error bar of stations' constraint violation.

We see that by increasing the number of vehicles of each type, the stations' expected demand will converge to actual demand, and coupling constraints will be satisfied. We consider 50 types of electrical vehicles. Each type has different number of EVs. Fig. 8 shows the error bar of the percentage of constraint violation to the average demand ratio. We see by increasing the number of EVs, this index converges to zero.

VI. Conclusion

We presented a non-cooperative game theoretical framework to study joint routing and destination selection problem of heterogeneous users by considering the variable pricing that depends on the demand on the station and the coupling constraints, which are concerned with the infrastructure and resource limitations. The application was demonstrated in EV public charging station. The proposed network flow model reduced the problem's complexity, as the users do not need to enumerate all feasible paths. The theory of variational inequality was exploited to prove the existence and uniqueness of the games' equilibrium and also to analyze the effect of coupling constraints. The proposed aggregative game framework resulted in scalability of the method, information privacy of the users, as well as decentralized solution for the Nash equilibrium seeking of the game through a projected gradient algorithm.

References

[1] D. Acemoglu, A. Makhdoumi, A. Malekian, and A. Ozdaglar, "Informational Braess' paradox: The effect of information on traffic congestion," *Oper. Res.*, vol. 66, no. 4, pp. 893–917, 2018.

[2] J. Thai, N. Laurent-Brouty, and A. M. Bayen, "Negative externalities of GPS-enabled routing applications: A game theoretical approach," in *Proc. IEEE 19th Int. Conf. Intell. Transp. Syst. (ITSC)*, Nov. 2016, pp. 595–601.

[3] X.-Y. Yan and T. Zhou, "Destination choice game: A spatial interaction theory on human mobility," *Sci. Rep.*, vol. 9, no. 1, pp. 1–9, Dec. 2019.

[4] M. Beliaev, E. Bıyık, D. A. Lazar, W. Z. Wang, D. Sadigh, and R. Pedarsani, "Incentivizing routing choices for safe and efficient transportation in the face of the COVID-19 pandemic," in *Proc. ACM/IEEE 12th Int. Conf. Cyber-Phys. Syst. (ICCPS)*, May 2021, pp. 187–197.

[5] *Coronavirus: Crowdless App Offers Shoppers Supermarket Crowd Levels*. Accessed: Mar. 1, 2021. [Online]. Available: https://www.bbc.com/news/uk-england-oxfordshire-52446658

[6] F. Rossi, R. Iglesias, M. Alizadeh, and M. Pavone, "On the interaction between autonomous mobility-on-demand systems and the power network: Models and coordination algorithms," *IEEE Trans. Control Netw. Syst.*, vol. 7, no. 1, pp. 384–397, Mar. 2020.

[7] N. Fulman and I. Benenson, "Establishing heterogeneous parking prices for uniform parking availability for autonomous and human-driven vehicles," *IEEE Intell. Transp. Syst. Mag.*, vol. 11, no. 1, pp. 15–28, Spring 2019.

[8] H. Lee and A. Clark, "Charging the future: Challenges and opportunities for electric vehicle adoption," HKS Fac. Res. Work. Paper Ser. RWP18-026, Sep. 2018.

[9] C. Liberto, G. Valenti, S. Orchi, M. Lelli, M. Nigro, and M. Ferrara, "The impact of electric mobility scenarios in large urban areas: The Rome case study," *IEEE Trans. Intell. Transp. Syst.*, vol. 19, no. 11, pp. 3540–3549, Nov. 2018.

[10] M. Alizadeh, H.-T. Wai, A. Goldsmith, and A. Scaglione, "Retail and wholesale electricity pricing considering electric vehicle mobility," *IEEE Trans. Control Netw. Syst.*, vol. 6, no. 1, pp. 249–260, Mar. 2019.

[11] A. Moradipari and M. Alizadeh, "Pricing and routing mechanisms for differentiated services in an electric vehicle public charging station network," *IEEE Trans. Smart Grid*, vol. 11, no. 2, pp. 1489–1499, Mar. 2020.

[12] B. Sohet, Y. Hayel, O. Beaude, and A. Jeandin, "Coupled charging-and-driving incentives design for electric vehicles in urban networks," *IEEE Trans. Intell. Transp. Syst.*, vol. 22, no. 10, pp. 6342–6352, Oct. 2021.

[13] F. He, Y. Yin, and S. Lawphongpanich, "Network equilibrium models with battery electric vehicles," *Transp. Res. B, Methodol.*, vol. 67, pp. 306–319, Sep. 2014.

[14] S. Manshadi, M. Khodayar, K. Abdelghany, and H. Üster, "Wireless charging of electric vehicles in electricity and transportation networks," *IEEE Trans. Smart Grid*, vol. 9, no. 5, pp. 4503–4512, Sep. 2017.

[15] A. Moradipari, N. Tucker, and M. Alizadeh, "Mobility-aware electric vehicle fast charging load models with geographical price variations," *IEEE Trans. Transport. Electrific.*, vol. 7, no. 2, pp. 554–565, Jun. 2021.

[16] S. R. Etesami, W. Saad, N. B. Mandayam, and H. V. Poor, "Smart routing of electric vehicles for load balancing in smart grids," *Automatica*, vol. 120, Oct. 2020, Art. no. 109148.

[17] C. Sun, X. Wen, Z. Lu, J. Zhang, and X. Chen, "A graphical game approach to electrical vehicle charging scheduling: Correlated equilibrium and latency minimization," *IEEE Trans. Intell. Transp. Syst.*, vol. 22, no. 1, pp. 505–517, Jan. 2021.

[18] W. Wei, L. Wu, J. Wang, and S. Mei, "Network equilibrium of coupled transportation and power distribution systems," *IEEE Trans. Smart Grid*, vol. 9, no. 6, pp. 6764–6779, Nov. 2018.

[19] W. Wei, W. U. Danman, W. U. Qiuwei, M. Shafie-Khah, and J. P. S. Catalão, "Interdependence between transportation system and power distribution system: A comprehensive review on models and applications," *J. Mod. Power Syst. Clean Energy*, vol. 7, no. 3, pp. 433–448, 2019.

[20] M. Zhu, X.-Y. Liu, and X. Wang, "Joint transportation and charging scheduling in public vehicle systems—A game theoretic approach," *IEEE Trans. Intell. Transp. Syst.*, vol. 19, no. 8, pp. 2407–2419, Aug. 2018.

[21] F. He, D. Wu, Y. Yin, and Y. Guan, "Optimal deployment of public charging stations for plug-in hybrid electric vehicles," *Transp. Res. B, Methodol.*, vol. 47, pp. 87–101, Jan. 2013.

[22] M. K. Jensen, "Aggregative games and best-reply potentials," *Econ. Theory*, vol. 43, no. 1, pp. 45–66, Apr. 2010.

[23] D. Paccagnan, B. Gentile, F. Parise, M. Kamgarpour, and J. Lygeros, "Nash and Wardrop equilibria in aggregative games with coupling constraints," *IEEE Trans. Autom. Control*, vol. 64, no. 4, pp. 1373–1388, Apr. 2019.

[24] H. Chen, Y. Li, R. H. Y. Louie, and B. Vucetic, "Autonomous demand side management based on energy consumption scheduling and instantaneous load billing: An aggregative game approach," *IEEE Trans. Smart Grid*, vol. 5, no. 4, pp. 1744–1754, Jul. 2014.

[25] J. Barrera and A. Garcia, "Dynamic incentives for congestion control," *IEEE Trans. Autom. Control*, vol. 60, no. 2, pp. 299–310, Feb. 2015.

[26] Z. Ma, D. S. Callaway, and I. A. Hiskens, "Decentralized charging control of large populations of plug-in electric vehicles," *IEEE Trans. Control Syst. Technol.*, vol. 21, no. 1, pp. 67–78, Jan. 2013.

[27] H. Kebriaei, S. J. Sadati-Savadkoohi, M. Shokri, and S. Grammatico, "Multi-population aggregative games: Equilibrium seeking via mean-field control and consensus," *IEEE Trans. Autom. Control*, early access, Feb. 4, 2021, doi: 10.1109/TAC.2021.3057063.

[28] S. Grammatico, "Dynamic control of agents playing aggregative games with coupling constraints," *IEEE Trans. Autom. Control*, vol. 62, no. 9, pp. 4537–4548, Sep. 2017.

[29] F. Parise, S. Grammatico, B. Gentile, and J. Lygeros, "Distributed convergence to Nash equilibria in network and average aggregative games," *Automatica*, vol. 117, Jul. 2020, Art. no. 108959.

[30] J. R. Correa, A. S. Schulz, and N. E. Stier-Moses, "Selfish routing in capacitated networks," *Math. Oper. Res.*, vol. 29, no. 4, pp. 961–976, 2004.

[31] S. H. Q. Li, Y. Yu, D. Calderone, L. Ratliff, and B. Açrkmeşe, "Tolling for constraint satisfaction in Markov decision process congestion games," in *Proc. Amer. Control Conf. (ACC)*, Jul. 2019, pp. 1238–1243.

[32] F. Facchinei and C. Kanzow, "Generalized Nash equilibrium problems," *Quart. J. Oper. Res.*, vol. 5, no. 3, pp. 173–210, Sep. 2007.

[33] F. Facchinei, A. Fischer, and V. Piccialli, "On generalized Nash games and variational inequalities," *Oper. Res. Lett.*, vol. 35, no. 2, pp. 159–164, 2007.

[34] *Traffic Assignment Manual*. Bur. Public Roads, U.S. Dept. Commerce, Urban Planning Division, Washington, DC, USA, 1964.

[35] Y. Lv, Y. Duan, W. Kang, Z. Li, and F.-Y. Wang, "Traffic flow prediction with big data: A deep learning approach," *IEEE Trans. Intell. Transp. Syst.*, vol. 16, no. 2, pp. 865–873, Apr. 2015.

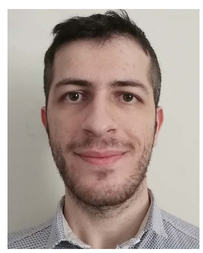
[36] F. Facchinei and J. Pang, *Finite-Dimensional Variational Inequalities and Complementarity Problems*. Berlin, Germany: Springer, 2007.

[37] D. Calderone, E. Mazumdar, L. J. Ratliff, and S. S. Sastry, "Understanding the impact of parking on urban mobility via routing games on queue-flow networks," in *Proc. IEEE 55th Conf. Decis. Control (CDC)*, Dec. 2016, pp. 7605–7610.

[38] D. P. Palomar and Y. C. Eldar, *Convex Optimization in Signal Processing and Communications*. Cambridge, U.K.: Cambridge Univ. Press, 2009, pp. 1–498.

[39] A. Auslender and M. Teboulle, "Lagrangian duality and related multiplier methods for variational inequality problems," *SIAM J. Optim.*, vol. 10, no. 4, pp. 1097–1115, Jan. 2000.

[40] *Average Price of Electricity to Ultimate Customers by End-Use Sector*. Accessed: Mar. 1, 2021. [Online]. Available: https://www.eia.gov/electricity/monthly/



Babak Ghaffarzadeh Bakhshayesh (Graduate Student Member) received the B.Sc. degree in electrical engineering from the University of Tabriz, Tabriz, Iran, in 2018, and the M.Sc. degree in electrical engineering from the University of Tehran, Tehran, Iran, in 2021. He is currently a Research Assistant with the Smart Networks Laboratory, University of Tehran. His research interests include the use of game theory, optimization, control, and learning in the design and analysis of complex societal-scale systems.

Hamed Kebriaei (Senior Member, IEEE) was born in 1983. He received the Ph.D. degree in control systems from the University of Tehran, Iran, in 2010. He has been the Head of the Control Department, School of ECE, University of Tehran, since 2017. He is currently an Associate Professor of control systems with the School of Electrical and Computer Engineering, University of Tehran. He has coauthored more than 70 research publications in reputed journals and conferences. His research interests include game theory, distributed optimization, and multi agent learning with applications in network systems. He is a Technical Committee Member of IEEE CSS in Networks and Communication Systems and a Board Member of Control Systems Chapter, IEEE Iran Section.