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Numerical Simulation of Flow over Non-Linearly Stretching Sheet Considering Chemical Reaction and Magnetic Field

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Abstract: The purpose of this paper is to investigate a system of differential equations related to the viscous flow over a stretching sheet. It is assumed that the intended environment for the flow includes a chemical reaction and a magnetic field. The governing equations are defined on the semi-finite domain and a numerical scheme, namely rational Gegenbauer collocation method is applied to solve it. In this method, the problem is solved in its main interval (semi-infinite domain) and there is no need to truncate it to a finite domain or change the domain of the problem. By carefully examining the effect of important physical parameters of the problem and comparing the obtained results with the answers of other methods, we show that despite the simplicity of the proposed method, it has a high degree of convergence and good accuracy.

Keywords: system of non-linear ODE; collocation method; rational Gegenbauer functions; non-linearly stretching sheet

1. Introduction

The problem of the boundary layer that arises on continuously stretching sheet is one of the important phenomena in engineering and industrial processes. For this reason, a lot of research has been done to address this issue. Sakiadis [1,2] was the first who studied numerically the boundary layer flows of a viscous fluid on surfaces that are continuously moving and their material is solid. Crane [3] considered the stretching sheet in this problem. The effect of heat transfer on this surface, which has a constant velocity, was studied experimentally by Tsou et al. [4]. The more general issue of the effect of suction or injection on the two components of heat transfer and mass transfer in the boundary layer at the stretching sheet on a fixed-velocity moving plate was investigated by Erickson et al. [5]. The chemical reaction was also studied in some research. For instance, the presence of a chemical reaction on the flow past an infinite vertical plate with uniform thermal flux, and its influence was considered in [6]. Anjalidevi and Kandasamy [7] considered the same problem in the presentation of heat transfer on the flow past a (semi-infinite) horizontal plate. They also expanded their assumptions by considering both heat transfer and magnetic field in [8] and reported the influence of a chemical reaction on the flow with these conditions. When considering the vertical plate with impulsive motion, the authors of [9,10], studied the effect of a chemical reaction on the unsteady flow passing through this plate subjected to uniform heat flux and uniform mass flux, respectively. Moreover, the influence of suction and a chemical reaction on heat and mass transfer along a moving vertical have been considered in [11]. Also, some numerical studies for the steady magnetohydrodynamic (MHD) non-Newtonian fluid was considered. Among them, we can mention to [12], which investigated the MHD power-law fluid flow over a motion plate by considering the diffusion of a kind of chemical reaction.

The purpose of this paper is to study the equations related to the boundary layer phenomenon of flow through a non-linear semi-infinite stretching sheet in the environment with a chemical reaction and a magnetic field. Some analytical methods like shooting method [13], Adomian decomposition method [14] and Homotopy analysis method [15] have been applied for solving this problem. Now, we are interested in applying the spectral method using an orthogonal system of functions, namely Gegenbauer polynomials, to obtain an approximate solution for this problem.

Spectral methods, which are the common numerical methods for solving differential type of equations, are considered a part of the general category of weighted residual methods (WRMs) [16]. The goal of the WRM as an approximation method is to minimize residuals or errors and the technique used in this direction leads to different methods such as collocation, Tau and Galerkin methods [17–19].

To solve many problems in physics and engineering, we have to first model them with some differential equations in unbounded domains or semi-infinite domains and then resolve them. One way to solve such equations is to use spectral methods. A straightforward approach is to use polynomials defined and be orthogonal over (semi) infinite domains like Hermite and Laguerre polynomials [20,21]. Another way is to change the problem interval to a finite domain and use functions defined in finite intervals such as Jacobi polynomials [22] in the spectral method. Another solution could be the domain truncation method [23]. But a direct method can be based on the use of rational approximations [24,25]. In this method, the systems of rational functions, which are mutually orthogonal on the semi-infinite domain, are constructed and spectral schemes are applied using these functions for solving equations on the half-line.

Authors of [26–31], applied the spectral method based on the rational Tau and the rational collocation approaches to solve the non-linear ordinary differential equations, which are defined on semi-infinite intervals. The collocation method is very popular due to its good accuracy and convergence, and the point to be considered in this method is to choose the appropriate basis functions. So, we use rational Gegenbauer functions as the basis function in this paper to solve a system of differential equations related to the viscous flow, which we mentioned above and will explain in detail in the next section. The rational Gegenbauer functions have some properties like easy computation, rapid convergence and completeness. By applying these functions, which have the property of orthogonality, to collocation method, any solution can be represented with arbitrary high accuracy while not need to reform the problem to a finite domain and this superiority gives the method a wider applicability. Actually, the rational Gegenbauer collocation method can be applied for solving other problems arising in science and engineering whose domain defined on the infinite domain and modelled by some differential equations such as PDEs system and fractional PDEs as well as integral equations. It is worth noting that using the rational Gegenbauer functions in collocation method can lead to better convergence rate by regulating extra parameter that makes the approach more accurate and flexible.

The rest of the article is divided as follows: the governing equations of a real problem and their corresponding parameters are described in Section 2. In Section 3 the properties of Gegenbauer polynomials and rational Gegenbauer functions are listed and how to apply rational Gegenbauer functions in collocation method is discussed. In Section 4, we apply collocation method to solve the problem. The numerical results and discussion about the effect of parameters are presented in Section 5 through some tables and figures. Finally, Section 6 makes concluding remarks.

2. Mathematical Formulation

The equations of the problem related to the steady two-dimensional incompressible flow of an electrically conducting viscous fluid through a non-linearly semi-infinite stretching sheet affected by chemical reaction and magnetic field is considered [13,14,32,33]:

$$\begin{aligned}
&\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,\\
&u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma {B_0}^2}{\rho}u,\\
&u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-k_1 C,
\end{aligned}\tag{1}$$

such that u and v are the velocity component along the x (coordinate along the sheet) and y (coordinate perpendicular to the sheet) directions, respectively. C is the concentration of species in the fluid. ν, σ and ρ are the kinematic viscosity, the electrical conductivity and the fluid density, respectively. The mass diffusion coefficient is D and the magnetic filed and chemical reaction parameters are B_0 and k_1, respectively [13,14]. The boundary conditions on the functions of this problem are:

$$\begin{aligned}
&u=ax+cx^2,\quad v=0,\quad C=C_w,\quad \text{at}\ \ y=0,\\
&u\to 0,\quad C\to 0,\quad \text{as}\ \ y\to\infty,
\end{aligned}\tag{2}$$

where w in C_w indicates wall condition and parameters a and c are constants.

Now, by introducing the similarity variables as follows, the governing PDEs reduced to ordinary ones [13,14,32,33]:

$$\begin{aligned}
&\eta=\left(\frac{a}{\nu}\right)^{1/2}y,\quad u=axf'(\eta)+cx^2g'(\eta),\\
&v=-(a\nu)^{1/2}f(\eta)-\frac{2cx}{(a/\nu)^{1/2}}g(\eta),\\
&C=C_w\left[C_0(\eta)+\frac{2cx}{a}C_1(\eta)\right],\\
&K=\frac{k_1 Sc}{a},\quad N=\frac{\sigma {B_0}^2}{a\rho},\quad Sc=\frac{\nu}{D}.
\end{aligned}\tag{3}$$

So, the following system of ODEs is obtained [13,14]:

$$\begin{cases}
f'''+ff''-f'^2-Nf'=0,\\
g'''+fg''-3f'g'+2f''g-Ng'=0,\\
C_0''+ScfC_0'-KC_0=0,\\
C_1''-Scf'C_1+ScgC_0'+ScfC_1'-KC_1=0,
\end{cases}\tag{4}$$

and the transformed boundary conditions are given by:

$$\begin{cases}
f(0)=0,\quad f'(0)=1,\quad f'(\infty)=0,\\
g(0)=0,\quad g'(0)=1,\quad g'(\infty)=0,\\
C_0(0)=1,\quad C_0(\infty)=0,\\
C_1(0)=0,\quad C_1(\infty)=0.
\end{cases}\tag{5}$$

Here, we have four functions, f, g, C_0, C_1 and the first two of which are relevant to the velocity fields and the last two are related to the concentrations of species in the fluid. The prime sign in these functions indicates the derivative of the function relative to η. Also, Sc is Schmidt number, K and N are reaction and magnetic parameters, respectively.

3. Rational Gegenbauer Collocation Method

In this section, we first explain Gegenbauer polynomials and rational Gegenbauer functions and examine the parameters and properties of each of them. Then, we explicate how a function defined in a semi-finite interval can be approximated by the rational Gegenbauer functions using the collocation method.

An initial definition for Gegenbauer polynomials (or ultraspherical polynomials) is as follows [34]:

$$G_n^{\alpha}(y) = \sum_{j=0}^{\lfloor n/2 \rfloor} (-1)^j \frac{\Gamma(n+\alpha-j)}{j!(n-2j)!\Gamma(\alpha)}(2y)^{n-2j}. \tag{6}$$

In this expression, there are two parameters n (an integer) and α (a real number $\geq -\frac{1}{2}$), which are the degree and order of Gegenbauer polynomials, respectively. Moreover, Γ is the Gamma function which is defined as $\Gamma(z) = \int_0^{\infty} t^{z-1}e^{-t}dt$.

Here, we study some properties of Gegenbauer polynomials [35]:

- They provide orthogonal polynomials for the interval $[-1, 1]$ due to the weight function $\rho(y) = (1-y^2)^{\alpha-\frac{1}{2}}$.
- These polynomials are obtained based on the following recursive formula:

$$\begin{cases} G_0^{\alpha}(y) = 1, \qquad G_1^{\alpha}(y) = 2\alpha y, \\ G_{n+1}^{\alpha}(y) = \frac{1}{n+1}\left[2y(n+\alpha)G_n^{\alpha}(y) - (n+2\alpha-1)G_{n-1}^{\alpha}(y)\right], \quad n \geq 1. \end{cases} \tag{7}$$

- The relationship between the derivative and Gegenbauer polynomials is as follows:

$$(G_n^{\alpha})'(y) = \frac{d}{dy}G_n^{\alpha}(y) = 2\alpha G_{n-1}^{\alpha+1}(y). \tag{8}$$

- The Gegenbauer polynomials are the special case of the Jacobi polynomials. Legendre and Chebyshev polynomials can also be reached by considering specific values for α parameter in Gegenbauer polynomials ($P_n(y) = G_n^{\frac{1}{2}}(y), T_n(\xi) = \frac{n}{2}\lim_{\alpha\to\infty}\frac{G_n^{\alpha}(y)}{\alpha}$ for $n \geq 1$, $U_n(y) = G_n^1(y)$, where P_n, T_n and U_n are Legendre polynomials, first and second kind of Chebyshev polynomials, respectively).

Now, we focus on the rational Gegenbauer functions which are obtained by considering $y = \frac{x-L}{x+L}$ as the input parameter of Gegenbauer polynomials. So, we have rational Gegenbauer functions which are denoted by $RG_n^{\alpha}(x) = G_n^{\alpha}(y)$ and are defined on the semi-infinite domain. Note that, here L is a constant scaling parameter [36] that with any choice of it, will have $y \in [-1, 1]$. The properties of these functions are listed as follows:

- They provide orthogonal functions for the interval $[0, \infty)$ due to the weight function $w(x) = \frac{2L}{(x+L)^2}\left[1-\left(\frac{x-L}{x+L}\right)^2\right]^{\alpha-\frac{1}{2}}$.
- These functions are obtained based on the following recursive formula:

$$\begin{cases} RG_0^{\alpha}(x) = 1, \qquad RG_1^{\alpha}(x) = 2\alpha\frac{x-L}{x+L}, \\ RG_{n+1}^{\alpha}(x) = \frac{1}{n+1}\left[2\left(\frac{x-L}{x+L}\right)(n+\alpha)RG_n^{\alpha}(x) - (n+2\alpha-1)RG_{n-1}^{\alpha}(x)\right], n \geq 1. \end{cases} \tag{9}$$

- The relationship between the derivative and the rational Gegenbauer functions is as follows:

$$(RG_n^\alpha)'(x) = \frac{d}{dx} RG_n^\alpha(x) = \frac{4\alpha L}{(x+L)^2} RG_{n-1}^{\alpha+1}(x). \tag{10}$$

- So, $(RG_n^\alpha)'(x)$ also are mutually orthogonal in $[0,\infty)$ by considering $w'(x) = \frac{(x+L)^2}{8L\alpha^2}\left[1-(\frac{x-L}{x+L})^2\right]^{\alpha+\frac{1}{2}}$ as the weight function.

  Consider $\zeta = [0,\infty)$ and define

$$L_w^2(\zeta) = \{v : \zeta \to \mathbb{R} \mid v \text{ is measurable and } \parallel v \parallel_w < \infty\}. \tag{11}$$

  Also, we know that the scalar product is defined as

$$< u, v >_w = \int_0^\infty u(x)v(x)w(x)dx, \tag{12}$$

which gives the norm

$$\parallel v \|_w = (\int_0^\infty \mid v(x) \mid^2 w(x)dx)^{\frac{1}{2}}. \tag{13}$$

As mentioned above, one of the most important features of $\{RG_n^\alpha(x)\}_{n\geq 0}$ is that they are orthogonal which can be obtained under Equation (12):

$$< RG_n^\alpha, RG_m^\alpha >_w = \frac{\pi 2^{1-2\alpha}\Gamma(n+2\alpha)}{n!(n+\alpha)\Gamma^2(\alpha)}\delta_{nm}, \tag{14}$$

where $\delta_{nm} = \begin{cases} 0 & n \neq m \\ 1 & n = m \end{cases}$ is known as the Kronecker delta function. In fact, this system is mutually orthogonal in $L_w^2(\zeta)$ and similarly, we have the following relation for the derivative of the system:

$$< (RG_n^\alpha)', (RG_m^\alpha)' >_{w'} = \frac{\pi 2^{-(2\alpha+1)}\Gamma(2\alpha+n+1)}{(n-1)!(n+\alpha)\Gamma^2(\alpha+1)}\delta_{nm}, \tag{15}$$

which indicates the mutually orthogonality of $(RG_n^\alpha)'(x)$ in $L_{w'}^2(\zeta)$.

Gauss integration was introduced in [37,38] and subsequently, Gauss points for rational Legendre and rational Chebyshev functions were presented in [39,40], respectively. So, one can have the rational Gegenbauer-Gauss interpolation, which is checked below.

The polynomial $G_{n+1}^\alpha(y)$ has $n+1$ roots which can be called Gegenbauer-Gauss points and can display with y_j, $j = 0, 1, \ldots, n$ and their corresponding Christoffel numbers are [34]:

$$\frac{2^{2-2\alpha}\pi\Gamma(n+1+2\alpha)}{(n+1)!\Gamma^2(\alpha)} \times \frac{1}{(1-y_j{}^2)[\frac{d}{dy}G_{n+1}^\alpha(y_j)]^2}. \tag{16}$$

Now, consider

$$x_j = L\frac{1+y_j}{1-y_j} \quad j = 0, 1, \ldots, n, \tag{17}$$

as the rational Gegenbauer-Gauss nodes which are the roots of $RG_{n+1}^\alpha(x)$. Considering Gauss integration in the semi-infinite domain and defining $\mathcal{RG}_n^\alpha = span\{RG_0^\alpha, RG_1^\alpha, ..., RG_n^\alpha\}$, cause to have:

$$\int_0^\infty u(x)w(x)dx = \int_{-1}^{1} u\left(L\frac{1+y}{1-y}\right)\rho(y)dy$$
$$= \sum_{j=0}^{N} u(x_j)w_j \quad \forall u \in \mathcal{RG}_{2n}^{\alpha}, \tag{18}$$

such that

$$w_j = \frac{2^{2-2\alpha}\pi\Gamma(n+1+2\alpha)}{(n+1)!\Gamma^2(\alpha)} \times \frac{L}{x_j(x_j+L)^2[\frac{d}{dx}RG_{n+1}^{\alpha}(x_j)]^2}, \tag{19}$$

are the weights related to the rational Gegenbauer-Gauss nodes. These weights can be achieved by considering Equations (16) and (17). Note that Equation (18) obtained based on the assumptions that $\rho(y) = (1-y^2)^{\alpha-\frac{1}{2}}$ and $y = \frac{x-L}{x+L}$ that make us have $w(x)\frac{dx}{dy} = \rho(y)$.

Now, we discuss how to approximate the function with the rational functions. The system of $\{RG_n^{\alpha}(x)\}_{n\geq 0}$ is complete in $L_w^2(\zeta)$. So, any function in $u \in L_w^2(\zeta)$ can be written as the following expansion by getting help from the functions of this system:

$$u(x) = \sum_{k=0}^{\infty} a_k RG_k^{\alpha}(x). \tag{20}$$

The expansion coefficients which are indicated by a_k's follow the following equations:

$$a_k = \frac{< u, RG_k^{\alpha} >_w}{\parallel RG_k^{\alpha} \parallel_w^2}. \tag{21}$$

One can approximate a function u which is defined on a semi-infinite interval and has smoothness property, by truncating the above expansion to n sentences and have

$$P_n u(x) = \sum_{k=0}^{n} a_k RG_k^{\alpha}(x). \tag{22}$$

$P_n u(x)$ is an element of $\mathcal{RG}_n^{\alpha}$ that according to Equations (12) and (13) can be considered as an orthogonal projection of u on $\mathcal{RG}_n^{\alpha}$. So, based on the orthogonality of rational Gegenbauer functions, the following statement holds for the difference of the approximated function with its real value (error of estimation) and rational Gegenbauer functions [41]

$$< P_n u - u, RG_i^{\alpha} >_w = 0 \quad \forall RG_i^{\alpha} \in \mathcal{RG}_n^{\alpha}. \tag{23}$$

Here, we can apply a collocation method by considering the expansion of the function and substituting it in the governing equations to form the residual function $\mathrm{Res}(x)$. The values of the unknown coefficients, which are a_k's, can then be calculated by placing the suitable chosen points in the residual function and equaling it to zero, while it must be ensured that the boundary conditions are also satisfied.

4. Solving the Problem by RGC Method

The method used in this section to solve Equation (4) with initial conditions given by Equation (5), is rational Gegenbauer collocation method, which can be denoted by the abbreviation RGC. For applying this method, we should expand $f(\eta), g(\eta), C_0(\eta)$ and $C_1(\eta)$ with P_n operator, as given in the following:

$$P_n f(\eta) = \sum_{k=0}^{n} a_k RG_k^{\alpha}(\eta), \quad P_n g(\eta) = \sum_{k=0}^{n} b_k RG_k^{\alpha}(\eta),$$
$$P_n C_0(\eta) = \sum_{k=0}^{n} c_k RG_k^{\alpha}(\eta), \quad P_n C_1(\eta) = \sum_{k=0}^{n} d_k RG_k^{\alpha}(\eta). \tag{24}$$

Now, we can construct the following four formulas means $\mathrm{Res}_1(\eta), \mathrm{Res}_2(\eta), \mathrm{Res}_3(\eta)$ and $\mathrm{Res}_4(\eta)$ as the residual functions for the equations in the governing system, Equation (4)

$$\begin{aligned}
\mathrm{Res}_1(\eta) &= \frac{d^3}{d\eta^3} P_n f(\eta) + P_n f(\eta) \frac{d^2}{d\eta^2} P_n f(\eta) - \left(\frac{d}{d\eta} P_n f(\eta)\right)^2 - N \frac{d}{d\eta} P_n f(\eta),\\
\mathrm{Res}_2(\eta) &= \frac{d^3}{d\eta^3} P_n g(\eta) + P_n f(\eta) \frac{d^2}{d\eta^2} P_n g(\eta) - 3\frac{d}{d\eta} P_n f(\eta) \frac{d}{d\eta} P_n g(\eta)\\
&+ 2\frac{d^2}{d\eta^2} P_n f(\eta) P_n g(\eta) - N \frac{d}{d\eta} P_n g(\eta),\\
\mathrm{Res}_3(\eta) &= \frac{d^2}{d\eta^2} P_n C_0(\eta) + Sc P_n f(\eta) \frac{d}{d\eta} P_n C_0(\eta) - K P_n C_0(\eta),\\
\mathrm{Res}_4(\eta) &= \frac{d^2}{d\eta^2} P_n C_1(\eta) - Sc \frac{d}{d\eta} P_n f(\eta) P_n C_1(\eta) + Sc P_n g(\eta) \frac{d}{d\eta} P_n C_0(\eta)\\
&+ Sc P_n f(\eta) \frac{d}{d\eta} P_n C_1(\eta) - K P_n C_1(\eta).
\end{aligned} \tag{25}$$

The equations for obtaining the coefficients a_k's, b_k's, c_k's and d_k's come from equalizing residual functions to zero at suitable points, which are rational Gegenbauer-Gauss points $(\eta_j, j = 1, 2, ..., n-1)$ and considering the boundary conditions that determined over three mentioned functions: (Note that two boundary conditions, $f'(\infty) = 0$ and $g'(\infty) = 0$, are already satisfied.)

$$\begin{cases}
\mathrm{Res}_1(\eta_j) = 0, \quad j = 1, 2, ..., n-1,\\
\mathrm{Res}_2(\eta_j) = 0, \quad j = 1, 2, ..., n-1,\\
\mathrm{Res}_3(\eta_j) = 0, \quad j = 1, 2, ..., n-1,\\
\mathrm{Res}_4(\eta_j) = 0, \quad j = 1, 2, ..., n-1,\\
P_n f(0) = 0, \quad \frac{d}{d\eta} P_n f(\eta)\Big|_{\eta=0} = 1,\\
P_n g(0) = 0, \quad \frac{d}{d\eta} P_n g(\eta)\Big|_{\eta=0} = 1,\\
P_n C_0(0) = 1, \quad \lim_{\eta\to\infty} P_n C_0(\eta) = 0,\\
P_n C_1(0) = 0, \quad \lim_{\eta\to\infty} P_n C_1(\eta) = 0.
\end{cases} \tag{26}$$

Given Equation (26), we actually get a set of non-linear equations whose number is equal to the number of unknown coefficients, $4(n+1)$, and solving this system with a numerical scheme such as Newton method leads to obtaining unknown coefficients.

5. Results and Discussions

In this paper, tables and figures show the results obtained by $n = 19$, $L = 5$ and $\alpha = 2$ in RGC method for solving the equations model presented in Section 2. A full description of the problem could be obtained by considering the effects of all parameters in Equation (4). Here, we consider the various values of magnetic parameter (N), Schmidt number (Sc) and chemical reaction (K) and report numerical values obtained by the proposed method for the mass transfer coefficients $C_0'(0)$ and $C_1'(0)$, while drawing graphs for $f'(\eta)$ and $g'(\eta)$ as the velocity profiles and C_0 and C_1 as the distributions of the concentrations. Moreover, we consider the ranges $0.6 \leq N \leq 1.5$, $0.24 \leq Sc \leq 0.8$ and $0.2 \leq K \leq 1.2$, in order to show the effect of the parameters well and to be able to compare our results with Homotopy analysis method (HAM) and Runge-Kutta methods presented in [15].

Tables 1–3 show the comparison of $f''(0)$, $C_0'(0)$ and $C_1'(0)$ obtained by the collocation method with the rational Gegenbaur as basis functions, HAM and the Runge-Kutta solution where $K = 0.2$, $Sc = 0.24$ and N has various values. Tables 4 and 5 present $C_0'(0)$ and $C_1'(0)$ for several values of Schmidt number, Sc, considering fixed values of 0.2 and 0.8 for the parameters K and N, respectively. To observe the effect of K as a variable parameter, we present Tables 6 and 7 for the case $N = 0.8$ and $Sc = 0.24$.

The influence of magnetic parameter, N, on the velocity profiles, f' and g', depicted in Figures 1 and 2, respectively. To show that the introduced method is highly accurate, we consider the case where the problem has an exact solution and compare it with our results in Figure 1. This case happens when we consider only the first equation in Equation (4) and the related conditions corresponding to f. So, the exact solution of this equation is $f(\eta) = \frac{1}{\sqrt{1+N}}\left(1 - e^{-\sqrt{1+N}\eta}\right)$, which will be simplified to the $f(\eta) = 1 - e^{-\eta}$, when considering the magnetic field to be ineffective.

Table 1. Comparison of RGC, HAM and Runge-Kutta solution of $f''(0)$ for various values of N while $Sc = 0.24$ and $K = 0.2$.

N	RGC	HAM [15]	Runge-Kutta [15]
0.6	−1.2649109	−1.2649340	−1.264911
0.8	−1.3416408	−1.3416240	−1.341640
1.0	−1.4142137	−1.4140170	−1.414213
1.5	−1.5811390	−1.5802313	−1.581113

Table 2. Comparison of RGC, HAM and Runge-Kutta solution of $C_0'(0)$ for various values of N while $Sc = 0.24$ and $K = 0.2$.

N	RGC	HAM [15]	Runge-Kutta [15]
0.6	−0.505908	−0.505910	−0.50590
0.8	−0.503817	−0.503601	−0.50381
1.0	−0.501969	−0.501943	−0.50191
1.5	−0.498136	−0.498112	−0.49813

Table 3. Comparison of RGC, HAM and Runge-Kutta solution of $C_1'(0)$ for various values of N while $Sc = 0.24$ and $K = 0.2$.

N	RGC	HAM [15]	Runge-Kutta [15]
0.6	−0.040368	−0.040063	−0.04035
0.8	−0.040271	−0.040320	−0.04022
1.0	−0.040080	−0.040070	−0.04004
1.5	−0.039358	−0.039380	−0.03937

Table 4. Comparison of RGC, HAM and Runge-Kutta solution of $C_0'(0)$ for various values of Sc while $N = 0.8$ and $K = 0.2$.

Sc	RGC	HAM [15]	Runge-Kutta [15]
0.24	−0.503817	−0.503601	−0.50381
0.4	−0.544214	−0.544218	−0.54429
0.6	−0.596601	−0.596620	−0.59659
0.8	−0.650026	−0.650062	−0.65001

Table 5. Comparison of RGC, HAM and Runge-Kutta solution of $C_1'(0)$ for various values of Sc while $N = 0.8$ and $K = 0.2$.

Sc	RGC	HAM [15]	Runge-Kutta [15]
0.24	−0.040271	−0.040320	−0.04022
0.4	−0.068083	−0.067382	−0.06795
0.6	−0.102323	−0.101437	−0.10221
0.8	−0.134901	−0.133846	−0.13469

Table 6. Comparison of RGC, HAM and Runge-Kutta solution of $C_0'(0)$ for various values of K while $N = 0.8$ and $Sc = 0.24$.

K	RGC	HAM [15]	Runge-Kutta [15]
0.2	−0.503817	−0.503601	−0.50381
0.8	−0.933538	−0.933541	−0.93358
1.0	−1.036524	−1.036527	−1.03652
1.2	−1.129919	−1.129936	−1.12992

Table 7. Comparison of RGC, HAM and Runge-Kutta solution of $C_1'(0)$ for various values of K while $N = 0.8$ and $Sc = 0.24$.

K	RGC	HAM [15]	Runge-Kutta [15]
0.2	−0.040271	−0.040320	−0.040220
0.8	−0.030673	−0.030445	−0.030667
1.0	−0.029105	−0.028971	−0.029101
1.2	−0.027825	−0.027672	−0.027822

Parameter N and velocity profiles are indirectly related to each other, i.e., with increasing N, velocity profiles decrease. Figure 2 shows the negligible effect of magnetic field on $g'(\eta)$.

The resulting graphs of $C_0(\eta)$ and $C_1(\eta)$ for different values of N are shown in Figures 3 and 4. The magnetic field growth is directly related to the growth of the species concentrations. Figures 5 and 6 show the solution of the functions C_0 and C_1 for different values of η when $N = 0.8$ and $K = 0.2$ for various Sc. By increasing the Schmidt number, both $C_0(\eta)$ and $C_1(\eta)$ decrease. The behavior of the chemical reaction parameter K for species concentrations are depicted in Figures 7 and 8, respectively. In these plots, we can see that by increasing the value of K, the concentration of $C_0(\eta)$ decreases and the concentration of $C_1(\eta)$ increases.

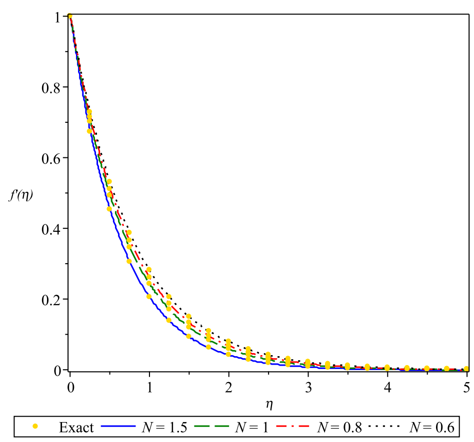

Figure 1. Dependency of velocity profile f' on the magnetic parameter N when $Sc = 0.24$ and $K = 0.2$ and comparison between the result of RGC method and the exact solution.

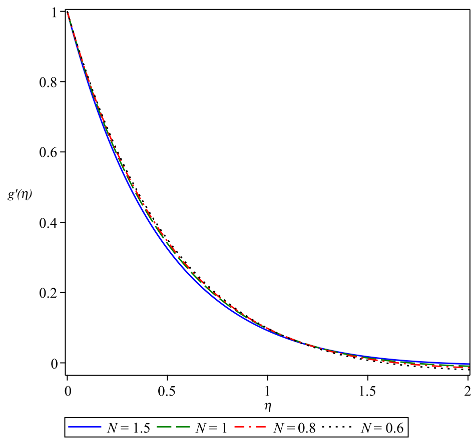

Figure 2. Dependency of velocity profile g' on the magnetic parameter N when $Sc = 0.24$ and $K = 0.2$.

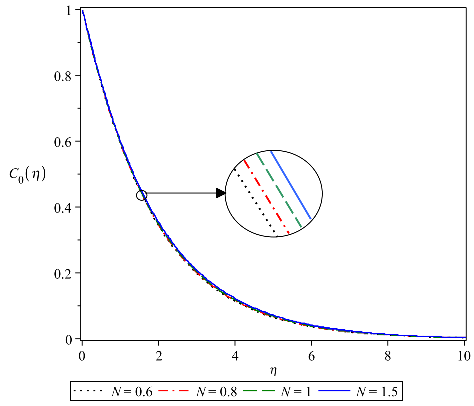

Figure 3. Dependency of concentration profile C_0 on the magnetic parameter N when $Sc = 0.24$ and $K = 0.2$.

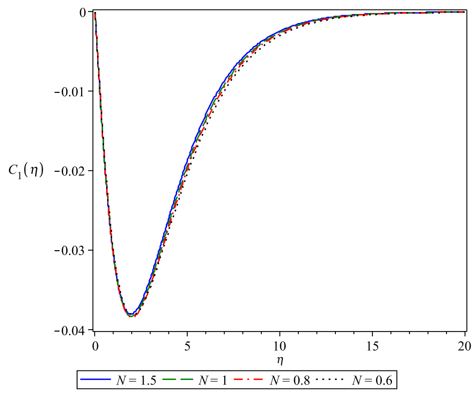

Figure 4. Dependency of concentration profile C_1 on the magnetic parameter N when $Sc = 0.24$ and $K = 0.2$.

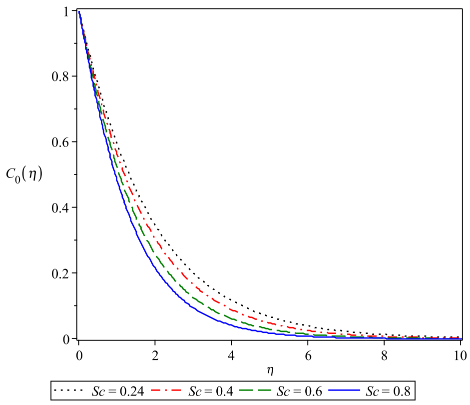

Figure 5. Dependency of concentration profile C_0 on the Schmidt number Sc when $N = 0.8$ and $K = 0.2$.

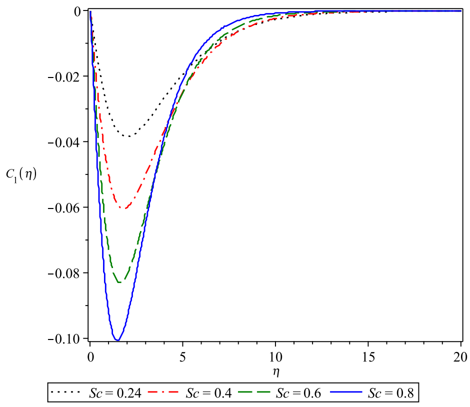

Figure 6. Dependency of concentration profile C_1 on the Schmidt number Sc when $N = 0.8$ and $K = 0.2$.

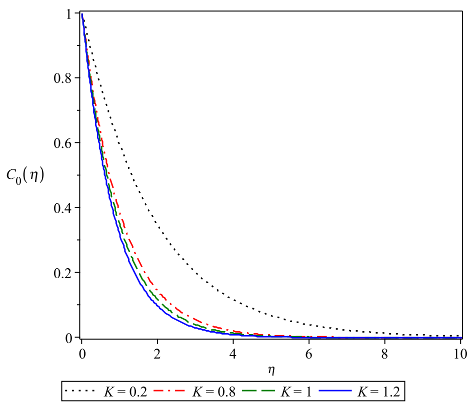

Figure 7. Dependency of concentration profile C_0 on the chemical reaction parameter K when $N = 0.8$ and $Sc = 0.24$.

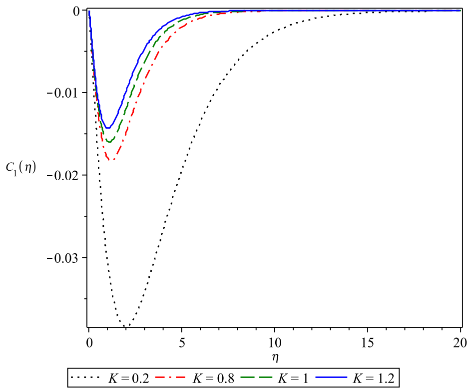

Figure 8. Dependency of concentration profile C_1 on the chemical reaction parameter K when $N = 0.8$ and $Sc = 0.24$.

In addition to the graphs related to the effect of the parameters, to check the accuracy of the proposed method, Figure 9 is drawn based on the norm of the residual functions. We define this norm ($\|\text{Res}\|^2$) as follows:

$$\|\text{Res}\|^2 = \int_0^\infty \text{Res}(\eta)^2\, d\eta. \tag{27}$$

This figure shows the logarithmic graph of the $\|\text{Res}_i\|^2$, $(i = 1, \ldots, 4)$, for $N = 0.8, K = 0.2$ and $Sc = 0.24$ that were obtained by various n in rational Gegenbauer collocation method. According to this diagram, it can be seen that with increasing the number of Gaussian points used in the method, the value of Equation (27) decreases for all four equations of the problem and the values reported in this descending diagram indicate the high accuracy of the method. Furthermore, the logarithmic graphs of absolute coefficients $|a_k|$, $|b_k|$, $|c_k|$ and $|d_k|$ of rational Gegenbauer functions in approximating the solutions for $n = 19$, are depicted in Figure 10. Descending orders of these graphs indicate the good convergence of the proposed method.

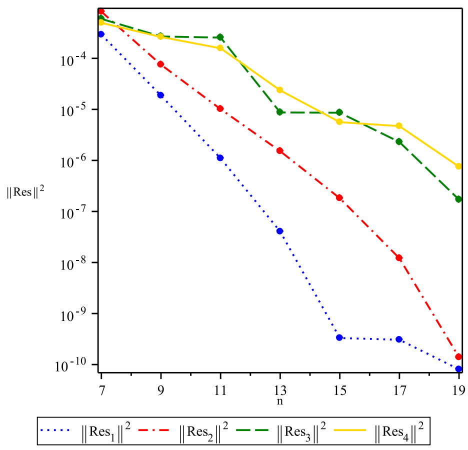

Figure 9. Logarithmic graphs of $\|\text{Res}_1\|^2$, $\|\text{Res}_2\|^2$, $\|\text{Res}_3\|^2$ and $\|\text{Res}_4\|^2$ by RGC method when $N = 0.8, K = 0.2$ and $Sc = 0.24$.

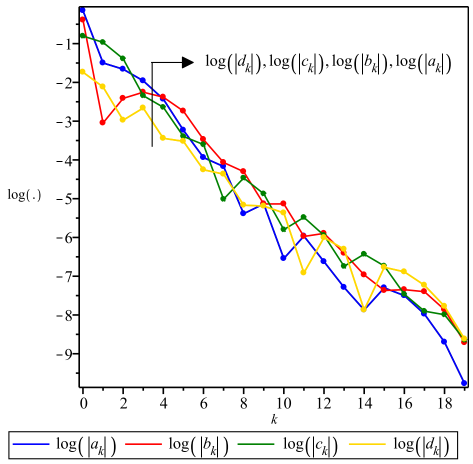

Figure 10. Logarithmic graphs of absolute coefficients $|a_k|$, $|b_k|$, $|c_k|$ and $|d_k|$ of the rational Gegenbauer functions when $N = 0.8, K = 0.2$ and $Sc = 0.24$.

6. Conclusions

In this paper, the problem of steady two-dimensional incompressible flow of an electrically conducting viscous fluid through a non-linearly semi-infinite stretching sheet affected by chemical reaction and magnetic field was considered. The method used to solve this problem was based on the rational Gegenbauer functions, which were integrated with the collocation method. The considered equations of the problem were defined in the semi-infinite intervals and we solved them without truncating them to the finite domains, due to the nature of the proposed method. This method results in good convergence and high accuracy. One of the benefits of using rational Gegenbauer functions is the ability to determine the α parameter, which can be selected according to the problem to get a better answer. The effects of magnetic parameter (N), chemical reaction parameter (K) and Schmidt number (Sc) on the velocity and concentration fields in the problem were presented through some tables and graphs.

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